Prince Edward Island
CANADA
Education and Early Years

Prince Edward Island Mathematics Curriculum

Mathematics

MAT621A

CURRICULUM

Prince Edward Island
CANADA

2013
Prince Edward Island
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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for Grades 10-12 Mathematics* (2008) has been adopted as the basis for a revised high school mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential Graduation Learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focused to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

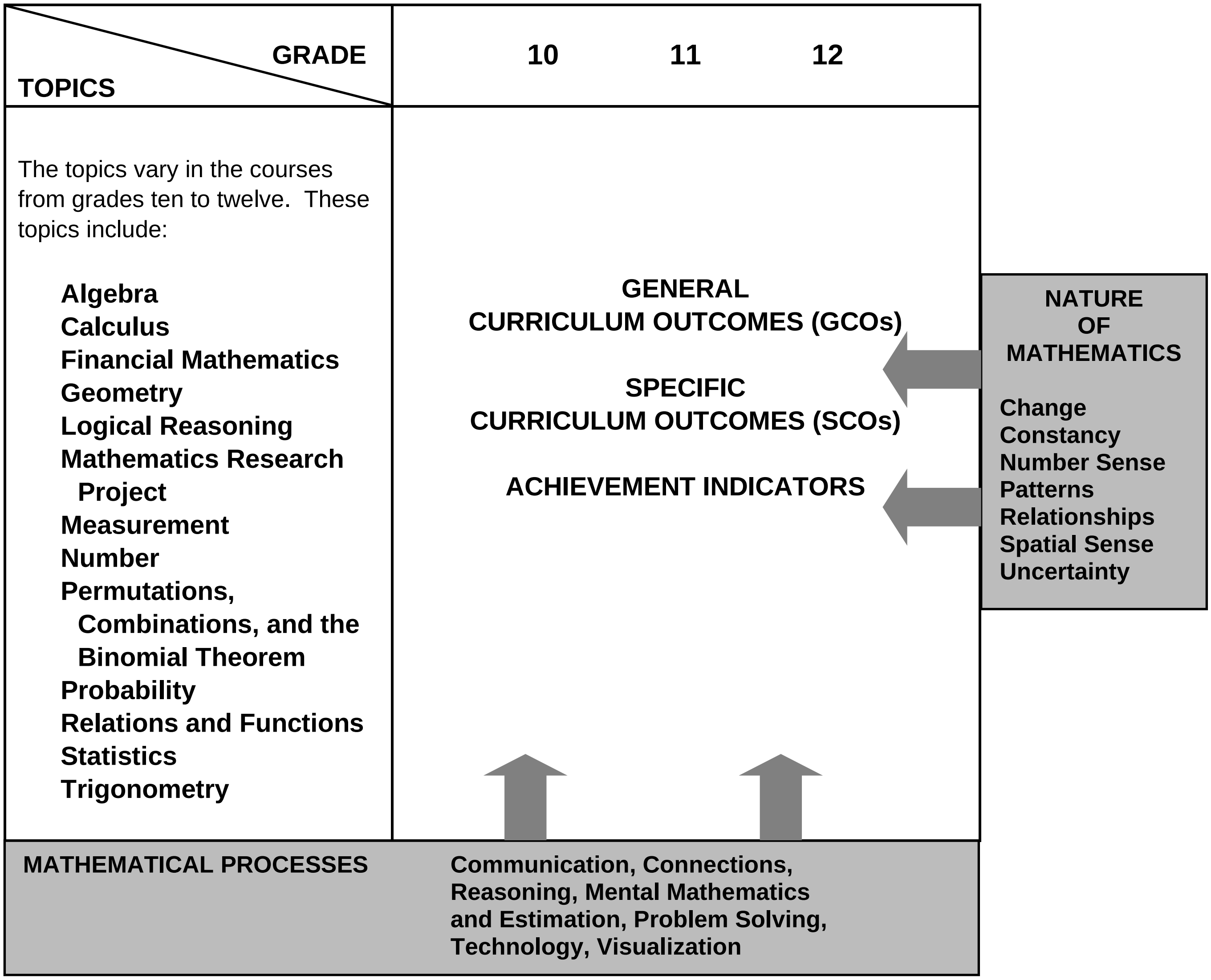
- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

➢ Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the understanding of mathematical concepts by students and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, physical education, and other subject areas. Efforts should be made to make connections and use examples drawn from a variety of disciplines.

Conceptual Framework for 10-12 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



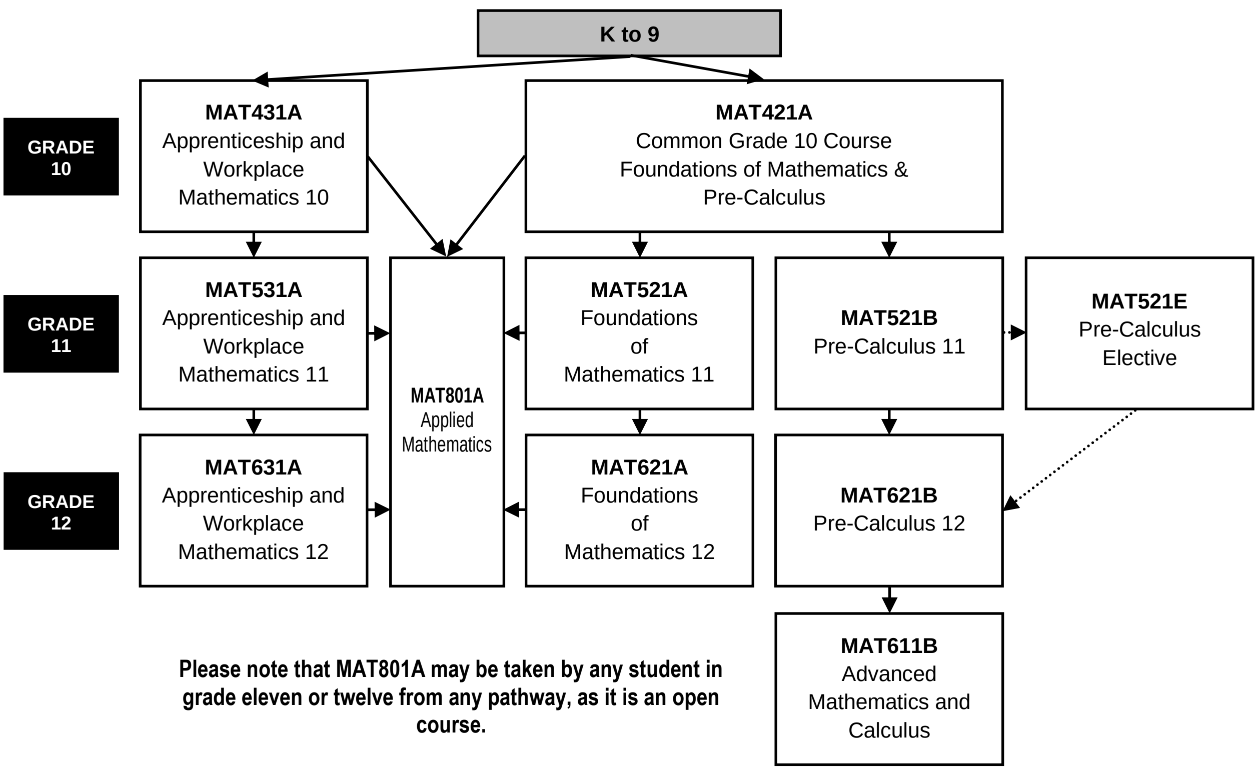
The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into a number of topics, as described above. These topics are not intended to be discrete units of instruction. The integration of outcomes across topics makes mathematical experiences meaningful. Students should make the connections among concepts both within and across topics. Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.

- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Pathways and Topics

The Prince Edward Island 10-12 mathematics curriculum includes pathways with corresponding topics rather than strands, which are found in the Prince Edward Island K-9 mathematics curriculum. Three pathways are available: **Apprenticeship and Workplace Mathematics**, **Foundations of Mathematics**, and **Pre-Calculus**. A common grade ten course (Foundations of Mathematics and Pre-Calculus, Grade 10) is the starting point for the Foundations of Mathematics pathway and the Pre-Calculus pathway. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings. These pathways are illustrated in the diagram below:



The goals of all three pathways are to provide the prerequisite knowledge, skills, understandings, and attitudes for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study or for direct entry into the work force. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Apprenticeship and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the work force. Topics include algebra, geometry, measurement, number, statistics, and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, algebra and number, logical reasoning, relations and functions, statistics, probability, and a mathematics research project.

Pre-Calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, combinatorics, and introductory calculus.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

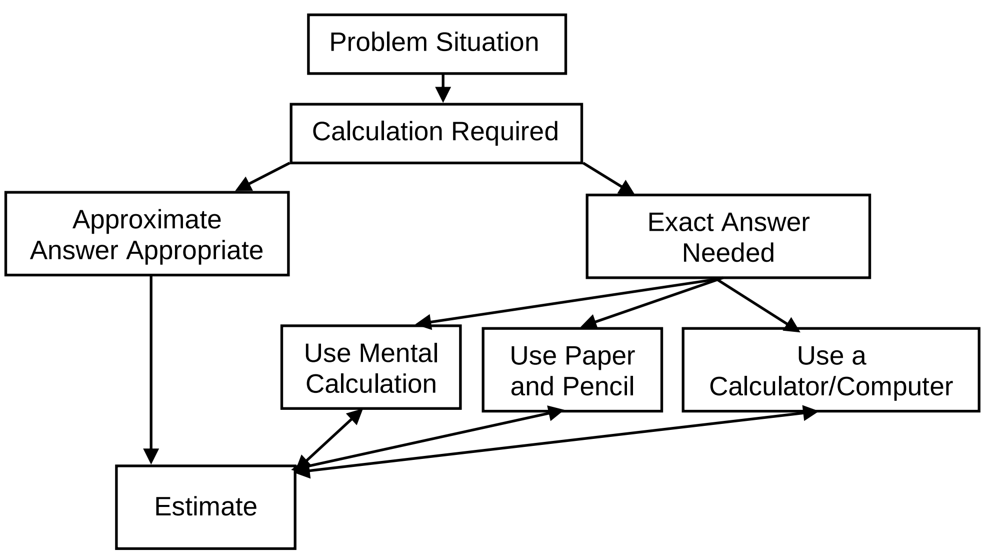
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001). Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you. . . ?" or "How could you. . . ?" the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;

- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.

- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all topics in mathematics and it is important that connections are made among topics. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2-D shapes and 3-D objects, and identify relationships to mathematical topics. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2-D shapes and 3-D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education" (p.60). The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate "communicating to learn mathematics and learning to communicate mathematically" (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database *Resources for Rethinking*, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

➢ Inquiry-Based Learning and Project Based Learning

Inquiry-based learning (IBL) allows students to explore, investigate, and construct new meaning from prior knowledge and from new information that is retrieved from other sources. It is not linear in nature, but promotes a continual looping back and forth throughout the process as students gather and process new information, redirect their inquiries, and continue through the process. Mathematical inquiry will require students to practise and refine their critical and creative-thinking skills. The terms *inquiry* and *research* are often used interchangeably within an educational context. While research often becomes the end-result of an inquiry process, it is the process itself that should be emphasized within an educational context. More information regarding the development of a mathematics research project is included in the appendix at the end of this document.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, teaching has been effective, or how best to address student learning needs. The quality of assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

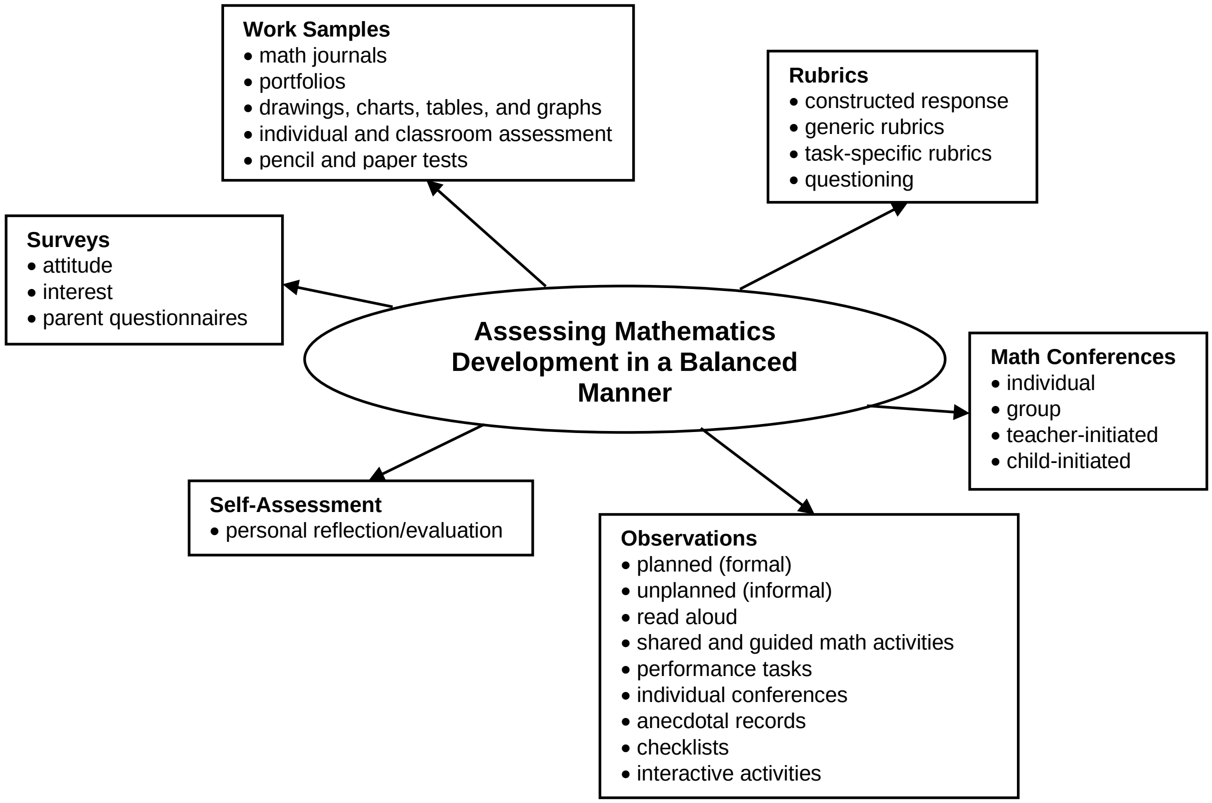
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children’s learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram on the next page.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;

- to provide the basis for sound decision-making about next steps in a student's learning.

➢ Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

➢ Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, phone calls, and electronic methods.

➢ Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;

- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he or she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island high school mathematics curriculum are organized into a number of topics across the grades from ten to twelve. Each topic has associated with it a general curriculum outcome (GCO). They are overarching statements about what students are expected to learn in each topic from grades ten to twelve.

Topic	General Curriculum Outcome (GCO)
Algebra (A)	Develop algebraic reasoning.
Algebra and Number (AN)	Develop algebraic reasoning and number sense.
Calculus (C)	Develop introductory calculus reasoning.
Financial Mathematics (FM)	Develop number sense in financial applications.
Geometry (G)	Develop spatial sense.
Logical Reasoning (LR)	Develop logical reasoning.
Mathematics Research Project (MRP)	Develop an appreciation of the role of mathematics in society.
Measurement (M)	Develop spatial sense and proportional reasoning. **(*Foundations of Mathematics and Pre-Calculus*)**
	Develop spatial sense through direct and indirect measurement. **(*Apprenticeship and Workplace Mathematics*)**
Number (N)	Develop number sense and critical thinking skills.
Permutations, Combinations and Binomial Theorem (PC)	Develop algebraic and numeric reasoning that involves combinatorics.
Probability (P)	Develop critical thinking skills related to uncertainty.
Relations and Functions (RF)	Develop algebraic and graphical reasoning through the study of relations.
Statistics (S)	Develop statistical reasoning.
Trigonometry (T)	Develop trigonometric reasoning.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific curriculum outcome.

In this curriculum guide, each specific curriculum outcome (SCO) is presented in a two-page format, and includes the following information:

- its corresponding topic and general curriculum outcome;
- the scope and sequence of the specific curriculum outcome(s) from grades eleven to twelve which correspond to this SCO;
- the specific curriculum outcome, with a list of achievement indicators;

- a list of the sections in *Foundations of Mathematics 12* which address the SCO, with specific achievement indicators highlighted in brackets;
- an elaboration for the SCO.

In the second half of this document, a curriculum guide supplement is presented which follows the primary resource, *Foundations of Mathematics 12*. As well, an appendix is included which outlines the steps to follow in the development of an effective mathematics research project.

FINANCIAL MATHEMATICS

SPECIFIC CURRICULUM OUTCOMES

FM1 – Solve problems that involve compound interest in financial decision making.

FM2 – Analyse costs and benefits of renting, leasing and buying.

FM3 – Analyse an investment portfolio in terms of:

- **interest rate;**
- **rate of return;**
- **total return.**

MAT621A – Topic: Financial Mathematics (FM)

GCO: Develop number sense in financial applications.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	FM1 Solve problems that involve compound interest in financial decision making.

SCO: **FM1 – Solve problems that involve compound interest in financial decision making. [C, CN, PS, T, V]**

Students who have achieved this outcome should be able to:

A. Solve problems that involve simple interest.

B. Explain the advantages and disadvantages of compound interest and simple interest.

C. Identify situations that involve compound interest.

D. Determine, given the principal, interest rate and number of compounding periods, the total interest of a loan.

E. Graph and compare, in a given situation, the total interest paid or earned for different compounding periods.

F. Determine the principal or present value of an investment, given the future value and compound interest rate.

G. Graph and describe the effects of changing the value of one of the variables in a situation that involves compound interest.

H. Determine, using technology, the total cost of a loan under a variety of conditions, *e.g.*, different amortization periods, interest rates, compounding periods and terms.

I. Compare and explain, using technology, different credit options that involve compound interest, including bank and store credit cards, and special promotions.

J. Solve a contextual problem that involves compound interest.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.1 (A)

1.2 (B C)

1.3 (D J)

1.4 (E F G J)

2.1 (H J)

2.2 (I J)

2.3 (I J)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: FM1 – Solve problems that involve compound interest in financial decision making. **[C, CN, PS, T, V]**

Elaboration

Simple interest is determined only on the principal of an investment. The value of an investment that earns simple interest over time is a linear function. The accumulated simple interest over time is also a linear function. Since the interest is paid at the end of each period, the growth is not continuous. The amount of simple interest earned on an investment can be determined using the formula

$$I = Prt$$

where I is the interest, P is the principal, r is the annual interest rate, expressed as a decimal, and t is the time, in years. The future value or amount, A, of an investment that earns simple interest can be determined using the formula

$$A = P(1+rt)$$

where P is the principal, r is the interest rate, expressed as a decimal, and t is the time, in years.

Compound interest is determined by applying the interest rate to the sum of the principal and any accumulated interest. Previously earned interest is reinvested over the course of the investment. If the same principal is invested in a compound interest account, with the same interest rate for the same term, the compound interest will grow faster (non-linear) than the corresponding simple interest (linear). The future value of an investment that earns compound interest can be determined using the compound interest formula

$$A = P(1+i)^n$$

where A is the future value, P is the principal, i is the interest rate per compounding period, expressed as a decimal, and n is the number of compounding periods. When using the compound interest formula, use an exact value for i. For example, for an annual interest rate of 5%, compounded monthly, use the exact value of $\frac{0.05}{12}$ for i instead of the decimal approximation 0.00416, when inputting values in a calculator.

Four common compounding frequencies are given in the table below. The table shows how to calculate the interest rate per compounding period (i) and the number of compounding periods (n) for each frequency.

COMPOUNDING FREQUENCY	TIMES PER YEAR	INTEREST RATE PER COMPOUNDING PERIOD (i)	NUMBER OF COMPOUNDING PERIODS (n)
annually	1	$i = \text{annual interest rate}$	$n = \text{number of years}$
semi-annually	2	$i = \frac{\text{annual interest rate}}{2}$	$n = 2 \times (\text{number of years})$
quarterly	4	$i = \frac{\text{annual interest rate}}{4}$	$n = 4 \times (\text{number of years})$
monthly	12	$i = \frac{\text{annual interest rate}}{12}$	$n = 12 \times (\text{number of years})$

The present value of an investment that earns compound interest can be determined using the formula

$$P = \frac{A}{(1+i)^n}$$

where P is the present value (or principal), A is the amount (or future value), i is the interest rate per compounding period, expressed as a decimal, and n is the number of compounding periods.

MAT621A – Topic: Financial Mathematics (FM)

GCO: Develop number sense in financial applications.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	FM2 Analyse costs and benefits of renting, leasing and buying.

SCO: FM2 – Analyse costs and benefits of renting, leasing and buying. [CN, PS, R, T]

Students who have achieved this outcome should be able to:

A. Identify and describe examples of assets that appreciate or depreciate.

B. Compare, using examples, renting, leasing and buying.

C. Justify, for a specific set of circumstances, if renting, leasing or buying would be advantageous.

D. Solve a problem involving renting, leasing or buying that requires the manipulation of a formula.

E. Solve, using technology, a contextual problem that involves cost-and-benefit analysis.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.4 (A B C D E)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: FM2 – Analyse costs and benefits of renting, leasing and buying. [CN, PS, R, T]

Elaboration

When deciding whether to rent, lease, or buy (with or without financing), each situation is unique. A cost and benefit analysis should take everything into account.

- Costs include initial costs and fees, short-term costs, long-term costs, disposable income, the cost of financing, depreciation and appreciation, penalties for breaking contracts, and equity.
- Benefits include convenience, commitments, flexibility, and personal needs or wants, such as how often a person wants to acquire a new car, for example.

Since each situation is unique, it is impossible to generalize about whether renting, leasing, or buying is best.

When renting, leasing or buying, payments often have to be made up front. Some payments go toward the overall cost, such as a down payment on a house, or a lease deposit and the first and last month's rent. Other deposits, such as a rental damage deposit, are refunded at a later date.

Appreciation and depreciation affect the value of a piece of property and should be considered when making decisions about renting, leasing, or buying, based on a particular situation. They are usually expressed as a rate per annum.

Equity can make buying a house a form of investment. This should also be considered when deciding to rent, lease, or buy.

MAT621A – Topic: Financial Mathematics (FM)

GCO: Develop number sense in financial applications.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	FM3 Analyse an investment portfolio in terms of: • interest rate; • rate of return; • total return.

SCO: **FM3 – Analyse an investment portfolio in terms of:**

- **interest rate;**
- **rate of return;**
- **total return.**

[ME, PS, R, T]

Students who have achieved this outcome should be able to:

A. Determine and compare the strengths and weaknesses of two or more portfolios.

B. Determine, using technology, the total value of an investment when there are regular contributions to the principal.

C. Graph and compare the total value of an investment with and without regular contributions.

D. Apply the Rule of 72 to solve investment problems, and explain the limitations of the rule.

E. Determine, using technology, possible investment strategies to achieve a financial goal.

F. Explain the advantages and disadvantages of long-term and short-term investment options.

G. Explain, using examples, why smaller investments over a longer term may be better than larger investments over a shorter term.

H. Solve an investment problem.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.3 (D)

1.5 (B C)

1.6 (E F G H)

2.1 (A)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: FM3 – Analyse an investment portfolio in terms of:

- **interest rate;**
- **rate of return;**
- **total return.**

[ME, PS, R, T]

Elaboration

The Rule of 72 is a simple strategy for estimating the doubling time of an investment. It is most accurate when the interest is compounded annually. For example, $1000 invested at 3% interest, compounded annually, will double in value in about $72 \div 3$, or 24 years; and $1000 invested at 6%, compounded annually, will double in about $72 \div 6$, or 12 years.

For an investment that involves a series of equal deposits or payments made at regular intervals, the future value is the sum of all the regular payments plus the accumulated interest. The future value of an investment involving regular payments can be found by determining the sum of all the future values of each regular payment

$$A = R(1+i)^0 + R(1+i)^1 + R(1+i)^2 + \cdots + R(1+i)^{n-1}$$

where A is the amount, or future value of the investment, R is the regular payment, i is the interest rate per compounding period, expressed as a decimal, and n is the number of compounding periods.

The future value of a single deposit has a greater future value than a series of regular payments of the same total amount. Small deposits over a long term can have a greater future value than large deposits over a short term because there is more time for compound interest to be earned.

An investment portfolio can be built from different types of investments, such as single payment investments (for example, Canada Savings Bonds and Guaranteed Investment Certificates) and investments involving regular payments. Some of these investments, such as Canada Savings Bonds, lock in money for specified periods of time, thus limiting access to the money, but offer higher interest rates. Other investments, such as savings accounts, are accessible at any time, but offer lower interest rates. Investments that involve greater principal amounts invested, or greater regular payment amounts when contracted tend to offer higher interest rates.

The rate of return is a useful measure for comparing investment portfolios. The factors that contribute to a larger return on an investment are time, interest rate, and compounding frequency. The longer that a sum of money is able to earn interest at a higher rate compounded more often, the more interest will be earned. For investments involving regular payments, the payment frequency is also a factor.

LOGICAL REASONING

SPECIFIC CURRICULUM OUTCOMES

LR1 – Analyse puzzles and games that involve numerical and logical reasoning, using problem-solving strategies.

LR2 – Solve problems that involve the application of set theory.

LR3 – Solve problems that involve conditional statements.

MAT621A – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
LR2 Analyse puzzles and games that involve spatial reasoning, using problem-solving strategies.	**LR1** Analyse puzzles and games that involve numerical and logical reasoning, using problem-solving strategies.

SCO: **LR1 – Analyse puzzles and games that involve numerical and logical reasoning, using problem-solving strategies.** **[CN, ME, PS, R]**

Students who have achieved this outcome should be able to:

A. Determine, explain and verify a strategy to solve a puzzle or to win a game; *e.g.*:
- guess and check;
- look for a pattern;
- make a systematic list;
- draw or model;
- eliminate possibilities;
- simplify the original problem;
- work backward;
- develop alternate approaches.

B. Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.

C. Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Note: It is intended this outcome be integrated throughout the course by using games and puzzles such as chess, Sudoku, Nim, logic puzzles, magic squares, Kakuro and cribbage.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

Integrated throughout the text.

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: LR1 – Analyse puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. [CN, ME, PS, R]

Elaboration

It is intended this outcome be integrated throughout the course by using games and puzzles such as chess, Sudoku, Nim, logic puzzles, magic squares, Kakuro and cribbage. In each case, the student should be able to describe the strategy used to win the game or solve the puzzle after having gone through the process.

MAT621A – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	LR2 Solve problems that involve the application of set theory.

SCO: LR2 – Solve problems that involve the application of set theory. [CN, PS, R, V]

Students who have achieved this outcome should be able to:

A. Understand sets and set notation.

B. Provide examples of the empty set, disjoint sets, subsets and universal sets in context, and explain the reasoning.

C. Organize information such as collected data and number properties, using graphic organizers, and explain the reasoning.

D. Explain what a specified region in a Venn diagram represents, using connecting words (and, or, not) or set notation.

E. Determine the elements in the complement of a set, and the intersection or the union of two sets.

F. Explain how set theory is used in applications such as Internet searches, database queries, data analysis, games and puzzles.

G. Identify and correct errors in a given solution to a problem that involves sets.

H. Solve a contextual problem that involves sets, and record the solution, using set notation.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.1 (A B C)

3.2 (D)

3.3 (E)

3.4 (F G H)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

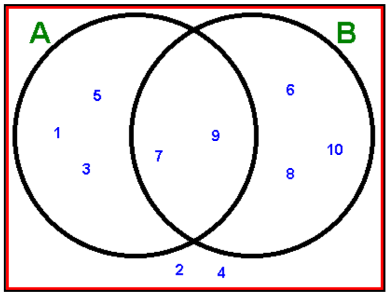
SCO: LR2 – Solve problems that involve the application of set theory. [CN, PS, R, V]

Elaboration

A set of elements can be represented by

- listing the elements; for example, $A = \{1, 2, 3, 4, 5\}$
- using words or a sentence; for example, $A = \{\text{all integers greater than 0 and less than 6}\}$
- using set builder notation, for example, $A = \{x \mid 0 < x < 6, x \in I\}$

The relationship between two sets can be illustrated by using a Venn diagram, as shown in the diagram at the right. As a rule, Venn diagrams do not usually show the relative sizes of the sets. The universal set can often be separated into subsets in more than one correct way.

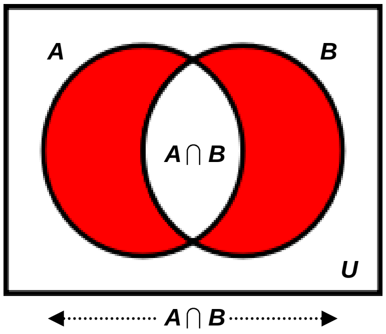


Sets are equal if they contain exactly the same elements, even if the elements are listed in different orders. Sometimes, it may not be possible to count the number of elements in very large or infinite sets, such as the set of real numbers.

The sum of the number of elements in a set, A, and its complement, A', is equal to the number of elements in the universal set, U.

$$n(A) + n(A') = n(U)$$

Each element in a universal set appears only once in a Venn diagram. If an element occurs in more than one set, it is placed in the area of the Venn diagram where the sets overlap. The union of two sets, denoted $A \cup B$, consists of all the elements that are in at least one of the sets. It is represented by the combined region of these sets on a Venn diagram. Union is indicated by the word *or*. The intersection of two sets, denoted $A \cap B$, consists of all the elements that are common to both sets. It is represented by the region of overlap on a Venn diagram. Intersection is indicated by the word *and*.



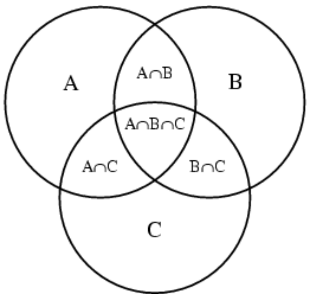
When two sets A and B are disjoint, the number of elements in A or B, denoted $n(A \cup B)$, is:

$$n(A \cup B) = n(A) + n(B)$$

If two sets, A and B, contain common elements, the number of elements in A or B, denoted $n(A \cup B)$, is:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Set theory is useful for solving many types of problems, including Internet searches, database queries, data analyses, games, and puzzles. To represent three intersecting sets with a Venn diagram, use three intersecting circles as shown in the diagram at the right. When solving a problem involving three sets, it is often best to start with the innermost section, where all three circles intersect, then work outward.



MAT621A – Topic: Logical Reasoning (LR)

GCO: Develop logical reasoning.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
LR1 Analyse and prove conjectures, using inductive and deductive reasoning.	**LR3** Solve problems that involve conditional statements.

SCO: **LR3 – Solve problems that involve conditional statements.** **[C, CN, PS, R]**

Students who have achieved this outcome should be able to:

A. Analyse an “if-then” statement, make a conclusion, and explain the reasoning.

B. Make and justify a decision, using “What if?” questions, in contexts such as probability, finance, sports, games or puzzles, with or without technology.

C. Determine the converse, inverse and contrapositive of an “if-then” statement; determine its veracity; and, if it is false, provide a counterexample.

D. Demonstrate, using examples, that the veracity of any statement does not imply the veracity of its converse or inverse.

E. Demonstrate, using examples, that the veracity of any statement does imply the veracity of its contrapositive.

F. Identify and describe contexts in which a biconditional statement can be justified.

G. Analyse and summarize, using a graphic organizer such as a truth table or Venn diagram, the possible results of given logical arguments that involve biconditional, converse, inverse or contrapositive statements.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.5 (A B C D F G)

3.6 (C D E G)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: LR3 – Solve problems that involve conditional statements. [C, CN, PS, R]

Elaboration

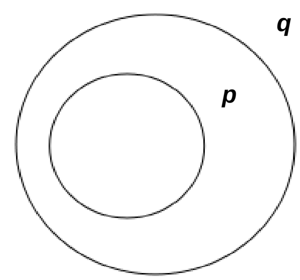
A conditional statement consists of a hypothesis, *p*, and a conclusion, *q*. Different ways to write a conditional statement include "If *p*, then *q*," "*p* implies *q*", and $p \rightarrow q$. To write the converse of a conditional statement, switch the hypothesis and the conclusion. In other words the converse of "If *p*, then *q*," is "If *q*, then *p*."

A conditional statement is either true or false. A truth table for a conditional statement, $p \rightarrow q$, can be set up as follows:

p	***q***	$p \rightarrow q$
T	T	T
F	F	T
F	T	T
T	F	F

A conditional statement is false only when the hypothesis is true and the conclusion is false. Otherwise, the conditional statement is true, even if the hypothesis is false.

A conditional statement can be represented using a Venn diagram with the inner oval representing the hypothesis and the outer oval representing the conclusion, as shown to the right. The statement "*p* implies *q*" means that *p* is a subset of *q*. Only one counterexample is needed to show that a conditional statement is false. If a conditional statement and its converse are both true, then they can be combined to create a biconditional statement using the phrase "if and only if."



The inverse of a conditional statement can be formed by negating the hypothesis and the conclusion. The contrapositive of a conditional statement can be formed by exchanging and negating the hypothesis and the conclusion. If a conditional statement is true, then its contrapositive is true, and vice versa. If the inverse of a conditional statement is true, then the converse of the statement is also true, and vice versa.

PROBABILITY

SPECIFIC CURRICULUM OUTCOMES

P1 – Interpret and assess the validity of odds and probability statements.

P2 – Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events.

P3 – Solve problems that involve the probability of two events.

P4 – Solve problems that involve the fundamental counting principle.

P5 – Solve problems that involve permutations.

P6 – Solve problems that involve combinations.

MAT621A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
S1 Demonstrate an understanding of normal distribution, including: • standard deviation; • *z*-scores.	**P1** Interpret and assess the validity of odds and probability statements.

SCO: **P1 – Interpret and assess the validity of odds and probability statements.** **[C, CN, ME]**

Students who have achieved this outcome should be able to:

A. Provide examples of statements of probability and odds found in fields such as media, biology, sports, medicine, sociology and psychology.

B. Explain, using examples, the relationship between odds (part-part) and probability (part-whole).

C. Express odds as a probability and vice versa.

D. Determine the probability of, or the odds for and against, an outcome in a situation.

E. Explain, using examples, how decisions may be based on probability or odds, and on subjective judgments.

F. Solve a contextual problem that involves odds or probability.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.1 (A)

5.2 (B C D E F)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: P1 – Interpret and assess the validity of odds and probability statements. [C, CN, ME]

Elaboration

An event is a collection of outcomes that satisfy a specific condition. For example, when throwing a regular die, the event “throw an odd number” is a collection of the outcomes 1, 3, and 5. The probability of an event can range from 0 (impossible) to 1 (certain). A probability can be expressed as a fraction, a decimal, or a percent. Theoretical probability can be used to determine the likelihood that an event will happen.

Knowing the probability of an event is useful when making decisions. The experimental probability of event *A* is represented as

$$P(A)=\frac{n(A)}{n(T)}$$

where $n(A)$ is the number of times event *A* occurred and $n(T)$ is the number of trials, *T*, in the experiment. The theoretical probability of event *A* is represented as

$$P(A)=\frac{n(A)}{n(S)}$$

where $n(A)$ is the number of favourable outcomes for event *A* and $n(S)$ is the total number of outcomes in the sample space, *S*, where all outcomes are equally likely. A game is considered fair when all of the players are equally likely to win.

Odds express a level of confidence about the occurrence of an event. The odds in favour of event *A* occurring are given by the ratio

$$\frac{P(A)}{P(A')} \text{ or } P(A):P(A')$$

This corresponds to the ratio of favourable outcomes to unfavourable outcomes. The odds against event *A* occurring are given by the ratio

$$\frac{P(A')}{P(A)} \text{ or } P(A'):P(A)$$

This corresponds to the ratio of unfavourable outcomes to favourable outcomes. In both expressions, $P(A')$ is the probability of the complement of *A*, where

$$P(A')=1-P(A)$$

If the odds in favour of event *A* occurring are $m:n$, then the odds against event *A* occurring are $n:m$. Finally, if the odds in favour of event *A* occurring are $m:n$, then

$$P(A)=\frac{m}{m+n}$$

MAT621A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
S1 Demonstrate an understanding of normal distribution, including: • standard deviation; • *z*-scores.	**P2** Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events.

SCO: **P2 – Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events.** **[CN, PS, R, V]**

Students who have achieved this outcome should be able to:

A. Classify events as mutually exclusive or non-mutually exclusive, and explain the reasoning.

B. Determine if two events are complementary, and explain the reasoning.

C. Represent, using set notation or graphic organizers, mutually exclusive (including complementary) and non-mutually exclusive events.

D. Solve a contextual problem that involves the probability of mutually exclusive or non-mutually exclusive events.

E. Solve a contextual problem that involves the probability of complementary events.

F. Create and solve a problem that involves mutually exclusive or non-mutually exclusive events.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.4 (A B C D E F)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: P2 – Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events. [CN, PS, R, V]

Elaboration

The favourable outcomes of two mutually exclusive events, *A* and *B*, can be represented as two disjoint sets. In this case, the probability that either *A* or *B* will occur is

$$P(A \cup B) = P(A) + P(B)$$

The favourable outcomes of two non-mutually exclusive events, *A* and *B*, can be represented as intersecting sets. In this case, the probability that either *A* or *B* will occur is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This is called the Principle of Inclusion and Exclusion. An alternate formula is

$$P(A \cup B) = P(A \setminus B) + P(B \setminus A) + P(A \cap B)$$

where $A \setminus B$ refers to the elements of *A* that are not in *B*, and $B \setminus A$ refers to the elements of *B* that are not in *A*. When two events are mutually exclusive, both results are equivalent, because $n(A \cap B) = 0$, which results in $P(A \cap B) = 0$.

MAT621A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
S1 Demonstrate an understanding of normal distribution, including: • standard deviation; • *z*-scores.	**P3** Solve problems that involve the probability of two events.

SCO: **P3 – Solve problems that involve the probability of two events. [CN, PS, R]**

Students who have achieved this outcome should be able to:

A. Compare, using examples, dependent and independent events.

B. Determine the probability of an event, given the occurrence of a previous event.

C. Determine the probability of two dependent or two independent events.

D. Create and solve a contextual problem that involves determining the probability of dependent or independent events.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.5 (A B C D)

5.6 (B C D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: P3 – Solve problems that involve the probability of two events. [CN, PS, R]

Elaboration

If the probability of one event depends on the probability of another event, then these events are called dependent events. For example, drawing a heart from a standard deck of 52 playing cards and then drawing another heart from the same deck without replacing the first card are dependent events. If event *B* depends on event *A* occurring, then the conditional probability that event *B* will occur, given that event *A* has occurred, can be represented by

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

If event *B* depends on event *A* occurring, then the probability that both events will occur can be represented by

$$P(A \cap B) = P(A) \bullet P(B \mid A)$$

If the probability of event *B* does not depend on the probability of event *A* occurring, then these events are called independent events. For example, tossing tails with a coin and drawing the ace of spades from a standard deck of 52 playing cards are independent events. The probability that two independent events, *A* and *B*, will both occur is the product of their individual probabilities:

$$P(A \cap B) = P(A) \bullet P(B)$$

A tree diagram is often used for modelling problems that involve both dependent and independent events.

MAT621A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	P4 Solve problems that involve the fundamental counting principle.

SCO: **P4 – Solve problems that involve the fundamental counting principle.** **[PS, R, V]**

Students who have achieved this outcome should be able to:

A. Represent and solve counting problems, using a graphic organizer.

B. Generalize the fundamental counting principle, using inductive reasoning.

C. Identify and explain assumptions made in solving a counting problem.

D. Solve a contextual counting problem, using the fundamental counting principle, and explain the reasoning.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.1 (A B C D)

4.7 (D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: P4 – Solve problems that involve the fundamental counting principle. [PS, R, V]

Elaboration

The Fundamental Counting Principle states that if one task can be performed in *a* ways, a second task can be performed in *b* ways, a third task in *c* ways, and so on, then all of these tasks can be performed in $a \bullet b \bullet c \bullet \cdots$ ways. The Fundamental Counting Principle applies when tasks are related by the word *and*.

The Fundamental Counting Principle does not apply when tasks are related by the word *or*. In this case, we apply one of the following formulas:

- If the tasks are mutually exclusive, they involve two disjoint sets, *A* and *B*, so $n(A \cup B) = n(A) + n(B)$.
- If the tasks are not mutually exclusive, they involve sets that are not disjoint, *C* and *D*, so $n(C \cup D) = n(C) + n(D) - n(C \cap D)$. In this case, the Principle of Inclusion and Exclusion must be used to avoid counting elements in the intersection of the two sets more than once.

Outcome tables, organized lists, and tree diagrams can also be used to solve counting problems. They have the added benefit of displaying all the possible outcomes, which can be useful in some problem situations. However, these strategies become difficult to use when there are many tasks involved and/or a large number of possibilities for each task.

MAT621A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	P5 Solve problems that involve permutations.

SCO: **P5 – Solve problems that involve permutations. [ME, PS, R, T, V]**

Students who have achieved this outcome should be able to:

A. Represent the number of arrangements of *n* elements taken *n* at a time, using factorial notation.

B. Determine, with or without technology, the value of a factorial.

C. Simplify a numeric or algebraic fraction containing factorials in both the numerator and denominator.

D. Solve an equation that involves factorials.

E. Determine the number of permutations of *n* elements taken *r* at a time.

F. Determine the number of permutations of *n* elements taken *n* at a time where some elements are not distinct.

G. Explain, using examples, the effect of the total number of permutations of *n* elements when two or more elements are identical.

H. Generalize strategies for determining the number of permutations of *n* elements taken *r* at a time.

I. Solve a contextual problem that involves probability and permutations.

Note: It is intended that circular permutations not be included.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.2 (A B C D)

4.3 (E H)

4.4 (F G)

4.7 (H)

5.3 (I)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: P5 – Solve problems that involve permutations. [ME, PS, R, T, V]

Elaboration

A permutation is an arrangement of objects in a definite order, where each object appears only once in each arrangement. For example, the three letters A, B, and C can be listed in 6 different ordered arrangements or permutations: ABC, ACB, BAC, BCA, CAB, CBA.

The expression $n!$ is called "*n* factorial" and represents the number of permutations of a set of *n* different objects. It is calculated as

$$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

The expression $n!$ is defined only for the values that belong to the set of whole numbers, that is, $n \in \{0,\ 1,\ 2,\ 3,\ ...\}$. If $n = 0,$ then $0!$ is defined as having a value of 1.

If order matters in a counting problem, then the problem involves permutations. The number of permutations from a set of *n* different objects, where *r* of then are used in each arrangement, can be calculated using the formula

$$_nP_r = \frac{n!}{(n-r)!},\ \text{where } 0 \le r \le n$$

When all *n* objects are used in each arrangement, *n* is equal to *r* and the number of arrangements is represented by $_nP_n = n!$. The number of permutations that can be created from a set of *n* objects, using *r* objects in each arrangement, where repetition is allowed, and $r \le n,$ is n^r. All of these formulas are based on the Fundamental Counting Principle.

If a counting problem has one or more conditions that must be met,

- first, consider each case that each condition creates, as the solution is developed;
- then, add the number of ways each case can occur to determine the total number of outcomes.

There are fewer permutations when some of the objects in a set are identical compared to when all the objects in a set are different. This is because some of the arrangements are identical. The number of permutations of *n* objects, where *a* are identical, another *b* are identical, another *c* are identical, and so on, is

$$P = \frac{n!}{a!b!c!\cdots}$$

Dividing $n!$ by $a!,\ b!,\ c!,$ and so on, deals with the effect of repetition caused by objects in the set that are identical. It eliminates arrangements that are the same and that would otherwise be counted multiple times.

MAT621A – Topic: Probability (P)

GCO: Develop critical thinking skills related to uncertainty.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
	P6 Solve problems that involve combinations.

SCO: P6 – Solve problems that involve combinations. [ME, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. Explain, using examples, why order is or is not important when solving problems that involve permutations or combinations.

B. Determine the number of combinations of n elements taken r at a time.

C. Generalize strategies for determining the number of combinations of n elements taken r at a time.

D. Solve a contextual problem that involves combinations and probability.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.5 (A)

4.6 (B C)

4.7 (C)

5.3 (D)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: P6 – Solve problems that involve combinations. [ME, PS, R, T, V]

Elaboration

When order does not matter in a counting problem, combinations are being determined. For example, ABC, ACB, BAC, BCA, CAB, and CBA are the six different permutations of the letters A, B, and C, but they all represent the same single combination of letters. When all of the objects are being used in a combination, there is only one possible combination.

The number of combinations from a set of n different objects, where only r of them are used in each combination, can be denoted by ${}_nC_r$ or $\binom{n}{r}$, and is calculated by using the formula

$${}_nC_r = \frac{n!}{r!(n-r)!}, \text{ where } 0 \le r \le n$$

The formula for ${}_nC_r$ is the formula for ${}_nP_r$ divided by $r!$. Dividing by $r!$ eliminates the counting of the same combinations of r objects arranged in different orders. From a set of n distinct objects, the number of combinations is always less than or equal to the number of permutations when selecting r of these objects, where $r \le n$. When solving problems involving combinations, it may also be necessary to use the Fundamental Counting Principle.

Sometimes combination problems that have conditions can be solved using direct reasoning. To do this, follow these steps:

- Consider only cases that reflect the conditions.
- Determine the number of combinations for each case.
- Add the results of the previous step to determine the total number of combinations.

Sometimes combination problems that have conditions can be solved using indirect reasoning. To do this, follow these steps:

- Determine the number of combinations without any conditions.
- Consider only cases that do not meet these conditions.
- Determine the number of combinations for each case identified in the previous step.
- From the number of combinations in the previous step, subtract the number of combinations without any conditions.

RELATIONS AND FUNCTIONS

SPECIFIC CURRICULUM OUTCOMES

RF1 – Represent data, using polynomial functions (of degree $\leq$ 3), to solve problems.

RF2 – Represent data, using exponential and logarithmic functions, to solve problems.

RF3 – Represent data, using sinusoidal functions, to solve problems.

MAT621A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
RF2 Demonstrate an understanding of the characteristics of quadratic functions, including: • vertex; • intercepts; • domain and range; • axis of symmetry.	**RF1** Represent data, using polynomial functions (of degree $\leq$ 3), to solve problems.

SCO: **RF1 – Represent data, using polynomial functions (of degree $\leq$ 3), to solve problems. [C, CN, PS, T, V]**

Students who have achieved this outcome should be able to:

A. Describe, orally and in written form, the characteristics of polynomial functions by analysing their graphs.

B. Describe, orally and in written form, the characteristics of polynomial functions by analysing their equations.

C. Match equations in a given set to their corresponding graphs.

D. Graph data and determine the polynomial function that best approximates the data.

E. Interpret the graph of a polynomial function that models a situation, and explain the reasoning.

F. Solve, using technology, a contextual problem that involves data that is best represented by graphs of polynomial functions, and explain the reasoning.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.1 (A)

6.2 (B C)

6.3 (D E F)

6.4 (D E F)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

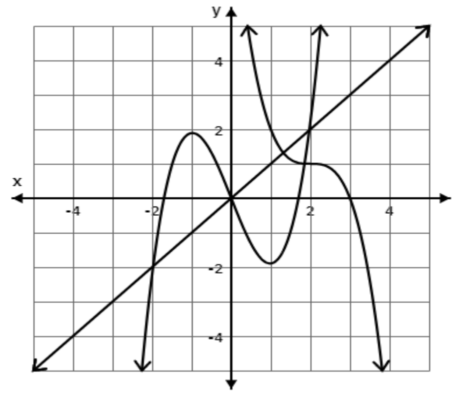
SCO: RF1 – Represent data, using polynomial functions (of degree $\le$ 3), to solve problems. [C, CN, PS, T, V]

Elaboration

A polynomial function in one variable is a function that contains only the operations of multiplication and addition, with real-number coefficients, whole-number exponents, and two variables. The degree of the function is the greatest exponent of the function. For example, $f(x) = 6x^3 + 3x^2 - 4x + 9$ is a cubic polynomial function of degree 3.

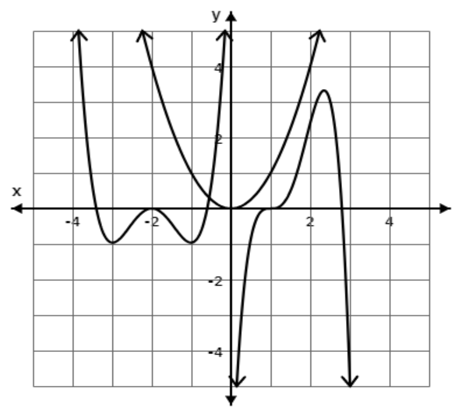
Graphs of odd degree have the following characteristics:

- a graph that extends down into Quadrant III and up into Quadrant I when the leading coefficient is positive
- a graph that extends up into Quadrant II and down into Quadrant IV when the leading coefficient is negative
- a *y*-intercept that corresponds to the constant term of the function
- at least one *x*-intercept and up to a maximum of *n* *x*-intercepts, where *n* is the degree of the function
- a domain of $\{x \mid x \in R\}$ and a range of $\{y \mid y \in R\}$
- no maximum or minimum points
- an even number of turning points



Graphs of even degree have the following characteristics:

- a graph that extends up into Quadrant II and up into Quadrant I when the leading coefficient is positive
- a graph that extends down into Quadrant III and down into Quadrant IV when the leading coefficient is negative
- a *y*-intercept that corresponds to the constant term of the function
- from zero to a maximum of *n* *x*-intercepts, where *n* is the degree of the function
- a domain of $\{x \mid x \in R\}$ and a range that depends on the maximum or minimum value of the function
- an odd number of turning points



A scatter plot is useful when looking for trends in a given set of data. If the points on a scatter plot seem to follow a linear, quadratic, or cubic trend, then there may be a polynomial relationship between the independent and the dependent variable. If the points on a scatter plot do follow a trend, technology can be used to determine the line or curve of best fit. The method used by technology to find the line or curve of best fit is called regression, which results in an equation that balances the points in the scatter plot on both sides of the line or curve.

A line or curve of best fit can be used to predict values that are not recorded or plotted. Predictions can be made by reading values from the line or curve of best fit on a scatter plot by using the equation of the line or curve of best fit.

MAT621A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
RF2 Demonstrate an understanding of the characteristics of quadratic functions, including: • vertex; • intercepts; • domain and range; • axis of symmetry.	**RF2** Represent data, using exponential and logarithmic functions, to solve problems.

SCO: **RF2 – Represent data, using exponential and logarithmic functions, to solve problems.**
[C, CN, PS, T, V]

Students who have achieved this outcome should be able to:

A. Describe, orally and in written form, the characteristics of exponential or logarithmic functions by analysing their graphs.

B. Describe, orally and in written form, the characteristics of exponential or logarithmic functions by analysing their equations.

C. Match equations in a given set to their corresponding graphs.

D. Graph data and determine the exponential or logarithmic function that best approximates the data.

E. Interpret the graph of an exponential or logarithmic function that models a situation, and explain the reasoning.

F. Solve, using technology, a contextual problem that involves data that is best represented by graphs of exponential or logarithmic functions, and explain the reasoning.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.1 (A)

7.2 (A B C)

7.3 (D E F)

7.4 (A B C)

7.5 (D E F)

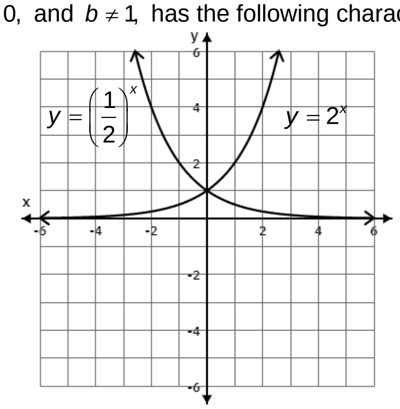
[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF2 – Represent data, using exponential and logarithmic functions, to solve problems.
[C, CN, PS, T, V]

Elaboration

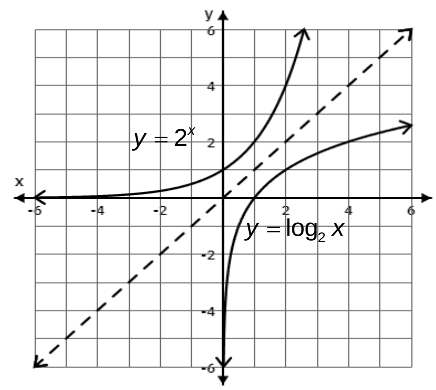
An exponential function of the form $y = a(b)^x$, where $a > 0$, $b > 0$, and $b \neq 1$, has the following characteristics:

- is increasing if $b > 1$
- is decreasing if $0 < b < 1$
- has a domain of $\{x \mid x \in R\}$
- has a range of $\{y \mid y > 0\}$
- has a y-intercept of 1
- has no x-intercept
- extends from Quadrant II to Quadrant I



The graphs of an exponential function and its inverse logarithmic function are reflections of each other in the line $y = x$, as shown below right. A logarithmic function of the form $y = \log_c x$, $c > 0$, $c \neq 1$, has the following characteristics:

- is increasing if $c > 1$
- is decreasing if $0 < c < 1$
- has a domain of $\{x \mid x > 0\}$
- has a range of $\{y \mid y \in R\}$
- has an x-intercept of 1
- has no y-intercept
- extends from Quadrant IV to Quadrant I if $c > 1$, or from Quadrant I to Quadrant IV if $0 < c < 1$



A scatter plot is useful when looking for trends in a given set of data. If the points on a scatter plot seem to follow an exponential or a logarithmic trend, then there may be an exponential or a logarithmic relationship between the independent and the dependent variable. If the points on a scatter plot do follow a trend, technology can be used to determine the curve of best fit. The method used by technology to find the curve of best fit is called regression, which results in an equation that balances the points in the scatter plot on both sides of the curve.

A curve of best fit can be used to predict values that are not recorded or plotted. Predictions can be made by reading values from the curve of best fit on a scatter plot by using the equation of the curve of best fit.

MAT621A – Topic: Relations and Functions (RF)

GCO: Develop algebraic and graphical reasoning through the study of relations.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
RF2 Demonstrate an understanding of the characteristics of quadratic functions, including: • vertex; • intercepts; • domain and range; • axis of symmetry.	**RF3** Represent data, using sinusoidal functions, to solve problems.

SCO: **RF3 – Represent data, using sinusoidal functions, to solve problems.** **[C, CN, PS, T, V]**

Students who have achieved this outcome should be able to:

A. Estimate and determine benchmarks for angle measure.

B. Describe, orally and in written form, the characteristics of sinusoidal functions by analysing their graphs.

C. Describe, orally and in written form, the characteristics of sinusoidal functions by analysing their equations.

D. Match equations in a given set to their corresponding graphs.

E. Graph data and determine the sinusoidal function that best approximates the data.

F. Interpret the graph of a sinusoidal function that models a situation, and explain the reasoning.

G. Solve, using technology, a contextual problem that involves data that is best represented by graphs of sinusoidal functions, and explain the reasoning.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

8.1 (A)

8.2 (B)

8.3 (B)

8.4 (C D)

8.5 (E F G)

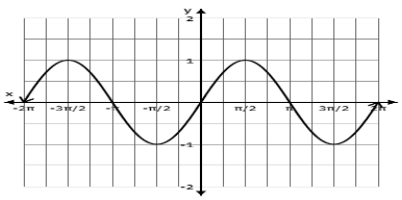
[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: RF3 – Represent data, using sinusoidal functions, to solve problems. [C, CN, PS, T, V]

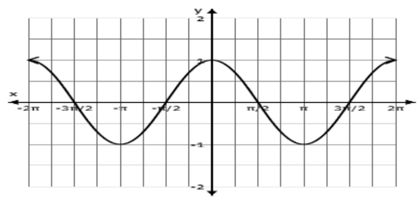
Elaboration

Radian measure is an alternative way to express the size of an angle. The measure of an angle can be expressed as a real number without units if radians are used. The central angle formed by one complete revolution in a circle is 360^0, or 2π, in radian measure. When converting between angle measures, 1 radian is equivalent to $\left(\frac{180}{\pi}\right)^0$ and 1^0 is equivalent to $\frac{\pi}{180}$ radians.

To sketch the graphs of $y = \sin x$ and $y = \cos x$, determine the coordinates of the key points representing the *x*-intercepts, maximum points and minimum points. Then, to get an accurate graph of the function, choose eight evenly-spaced points in each period of the function, and graph the results.



$y = \sin x$



$y = \cos x$

The following table highlights the characteristics of the graphs of each function:

	$y = \sin x$	$y = \cos x$
Maximum Value	1	1
Minimum Value	–1	–1
Amplitude	1	1
Period	2π or 360^0	2π or 360^0
***x*-intercepts**	$\pm\pi n,\ n \in I$	$\frac{\pi}{2} \pm \pi n,\ n \in I$
***y*-intercept**	0	1
Domain	$\{x \mid x \in R\}$	$\{x \mid x \in R\}$
Range	$\{y \mid -1 \le y \le 1\}$	$\{y \mid -1 \le y \le 1\}$
Midline	$y = 0$	$y = 0$

The characteristics of sinusoidal functions of the form $y = a \sin b(x-c)+d$ and $y = a \cos b(x-c)+d$ can be summarized as follows:

- The amplitude is represented by $|a|$. It can be found by using the formula $\text{amplitude} = \frac{\max - \min}{2}$.
- The period is can be calculated by using the formula $\text{period} = \frac{2\pi}{|b|}$, in radians, or $\text{period} = \frac{360^0}{|b|}$, in degrees.
- The horizontal translation is represented by *c*. It is a shift to the right if $c > 0$, and to the left if $c < 0$.
- The vertical displacement is represented by *d*. It is a shift up if $d > 0$, and a shift down if $d < 0$. As a result, the equation of the midline is $y = d$.

MATHEMATICS RESEARCH PROJECT

SPECIFIC CURRICULUM OUTCOMES

MRP1 – Research and give a presentation on a current event or an area of interest that involves mathematics.

MAT521A – Topic: Mathematics Research Project (MRP)

GCO: Develop an appreciation of the role of mathematics in society.

GRADE 11 – MAT521A	GRADE 12 – MAT621A
MRP1 Research and give a presentation on a historical event or an area of interest that involves mathematics.	**MRP1** Research and give a presentation on a current event or an area of interest that involves mathematics.

SCO: **MRP1 – Research and give a presentation on a current event or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]**

Students who have achieved this outcome should be able to:

A. Collect primary or secondary data (statistical or informational) related to the topic.

B. Assess the accuracy, reliability and relevance of the primary and secondary data collected by:
- identifying examples of bias and points of view;
- identifying and describing the data collection methods;
- determining if the data is relevant;
- determining if the data is consistent with information obtained from other sources on the same topic.

C. Interpret data, using statistical methods if applicable.

D. Identify controversial issues, if any, and present multiple sides of the issues with supporting data.

E. Organize and present the research project, with or without technology.

Section(s) in Foundations of Mathematics 12 *that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

Integrated throughout the text.

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: MRP1 – Research and give a presentation on a current event or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]

Elaboration

See the appendix at the end of this document for specific details regarding the development of a mathematics research project.

REFERENCES

American Association for the Advancement of Science [AAAS-Benchmarks]. *Benchmark for Science Literacy*. New York, NY: Oxford University Press, 1993.

Banks, James A. and Cherry A. McGee Banks. *Multicultural Education: Issues and Perspectives*. Boston: Allyn and Bacon, 1993.

Black, Paul and Dylan Wiliam. "Inside the Black Box: Raising Standards Through Classroom Assessment." *Phi Delta Kappan*, 20, October 1998, pp.139-148.

Canavan-McGrath, Cathy, *et. al.* *Foundations of Mathematics 11*. Nelson Education, 2012.

British Columbia Ministry of Education. *The Primary Program: A Framework for Teaching*, 2000.

Davies, Anne. *Making Classroom Assessment Work*. British Columbia: Classroom Connections International, Inc., 2000.

Hope, Jack A., *et. al*. *Mental Math in the Primary Grades*. Dale Seymour Publications, 1988.

National Council of Teachers of Mathematics. *Mathematics Assessment: A Practical Handbook*. Reston, VA: NCTM, 2001.

National Council of Teachers of Mathematics. *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000.

Rubenstein, Rheta N. *Mental Mathematics Beyond the Middle School: Why? What? How?* September 2001, Vol. 94, Issue 6, p. 442.

Shaw, Jean M. and Mary Jo Puckett Cliatt. "Developing Measurement Sense." In P.R. Trafton (ed.), *New Directions for Elementary School Mathematics* (pp. 149–155). Reston, VA: NCTM, 1989.

Steen, Lynn Arthur (ed.). *On the Shoulders of Giants – New Approaches to Numeracy.* Washington, DC: National Research Council, 1990.

Van de Walle, John A. and Louann H. Lovin. *Teaching Student-Centered Mathematics, Grades 5-8.* Boston: Pearson Education, Inc. 2006.

Western and Northern Canadian Protocol. *Common Curriculum Framework for 10-12 Mathematics*, 2008.