Prince Edward Island
CANADA

Education and Early Years

Prince Edward Island Mathematics Curriculum

CURRICULUM

Mathematics

Grade 9

Acknowledgments

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Background and Rationale

The development of an effective mathematics curriculum has encompassed a solid research base. Developers have examined the curriculum proposed throughout Canada and secured the latest research in the teaching of mathematics, and the result is a curriculum that should enable students to understand and use mathematics.

The Western and Northern Canadian Protocol (WNCP) *Common Curriculum Framework for K-9 Mathematics* (2006) has been adopted as the basis for a revised mathematics curriculum in Prince Edward Island. The *Common Curriculum Framework* was developed by the seven Canadian western and northern ministries of education (British Columbia, Alberta, Saskatchewan, Manitoba, Yukon Territory, Northwest Territories, and Nunavut) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP, and on the *Principles and Standards for School Mathematics* (2000), published by the National Council of Teachers of Mathematics (NCTM).

➢ Essential Graduation Learnings

Essential graduation learnings (EGLs) are statements describing the knowledge, skills, and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills, and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work, and study today and in the future. Essential graduation learnings are cross curricular, and curriculum in all subject areas is focussed to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Specifically, graduates from the public schools of Prince Edward Island will demonstrate knowledge, skills, and attitudes expressed as essential graduation learnings, and will be expected to

- respond with critical awareness to various forms of the arts, and be able to express themselves through the arts;
- assess social, cultural, economic, and environmental interdependence in a local and global context;
- use the listening, viewing, speaking, and writing modes of language(s), and mathematical and scientific concepts and symbols, to think, learn, and communicate effectively;
- continue to learn and to pursue an active, healthy lifestyle;
- use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts;
- use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

More specifically, curriculum outcome statements articulate what students are expected to know and be able to do in particular subject areas. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

➢ Curriculum Focus

There is an emphasis in the Prince Edward Island mathematics curriculum on particular key concepts at each grade which will result in greater depth of understanding. There is also more emphasis on number sense and operations in the early grades to ensure students develop a solid foundation in numeracy. The intent of this document is to clearly communicate to all educational partners high expectations for students in mathematics education. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM, *Principles and Standards for School Mathematics*, 2000).

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems;
- communicate and reason mathematically;
- appreciate and value mathematics;
- make connections between mathematics and its applications;
- commit themselves to lifelong learning;
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will

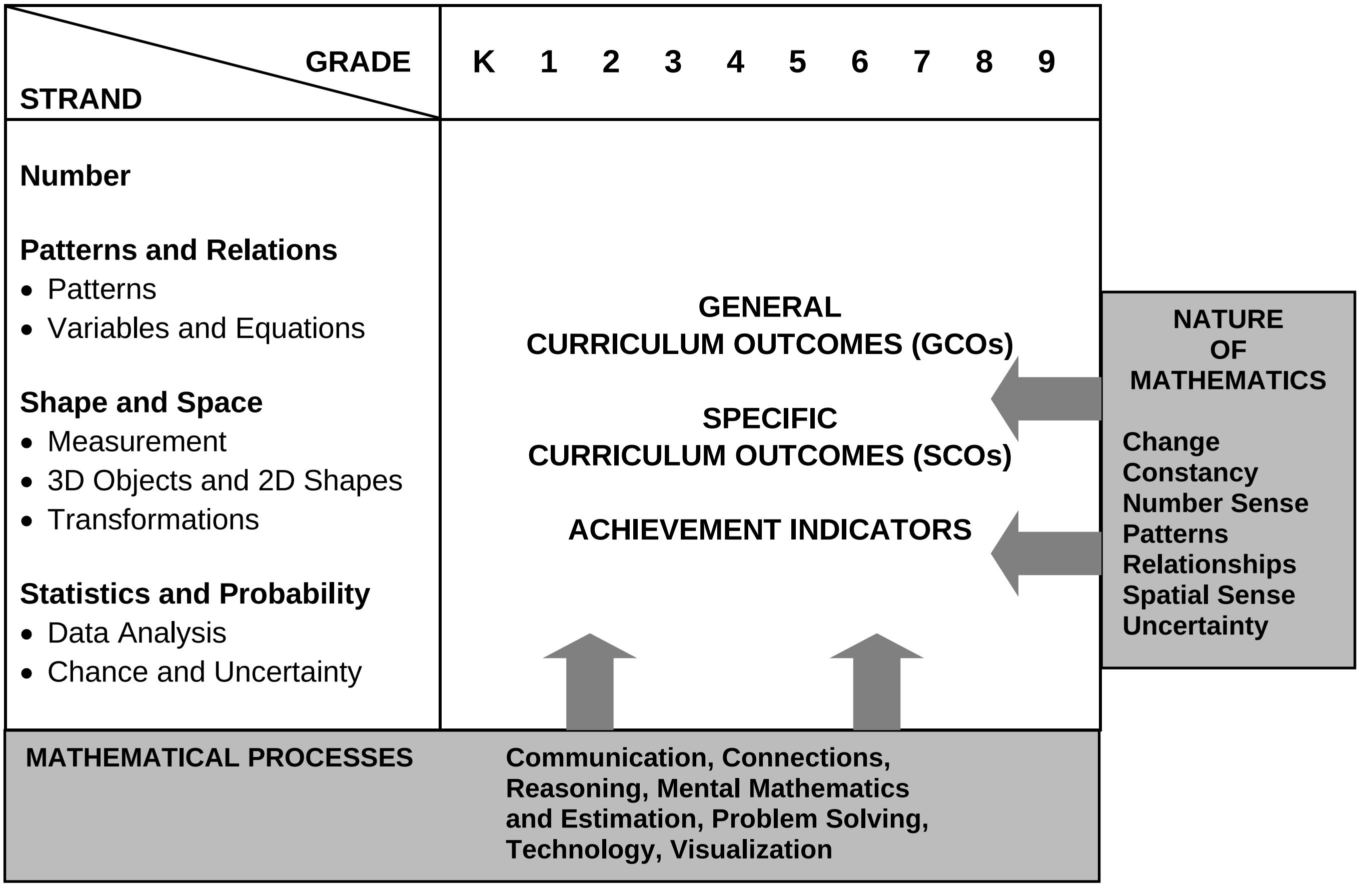
- gain understanding and appreciation of the contributions of mathematics as a science, philosophy, and art;
- exhibit a positive attitude toward mathematics;
- engage and persevere in mathematical tasks and projects;
- contribute to mathematical discussions;
- take risks in performing mathematical tasks;
- exhibit curiosity.

➢ Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the understanding of mathematical concepts by students and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, physical education, and other subject areas. Efforts should be made to make connections and use examples drawn from a variety of disciplines.

Conceptual Framework for K-9 Mathematics

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



The mathematics curriculum describes the nature of mathematics, as well as the mathematical processes and the mathematical concepts to be addressed. This curriculum is arranged into four strands, namely Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are not intended to be discrete units of instruction. The integration of outcomes across strands makes mathematical experiences meaningful. Students should make the connections among concepts both within and across strands. Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- Decreasing emphasis on rote calculation, drill, and practice, and the size of numbers used in paper and pencil calculations makes more time available for concept development.
- Problem solving, reasoning, and connections are vital to increasing mathematical fluency, and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using models and gradually developed from the concrete to the pictorial to the symbolic.

➢ Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics. The Prince Edward Island mathematics curriculum incorporates the following seven interrelated mathematical processes that are intended to permeate teaching and learning. These unifying concepts serve to link the content to methodology.

Students are expected to

- communicate in order to learn and express their understanding of mathematics; **[Communications: C]**
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines; **[Connections: CN]**
- demonstrate fluency with mental mathematics and estimation; **[Mental Mathematics and Estimation: ME]**
- develop and apply new mathematical knowledge through problem solving; **[Problem Solving: PS]**
- develop mathematical reasoning; **[Reasoning: R]**
- select and use technologies as tools for learning and solving problems; **[Technology: T]**
- develop visualization skills to assist in processing information, making connections, and solving problems. **[Visualization: V]**

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology. Communication can help students make connections among concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

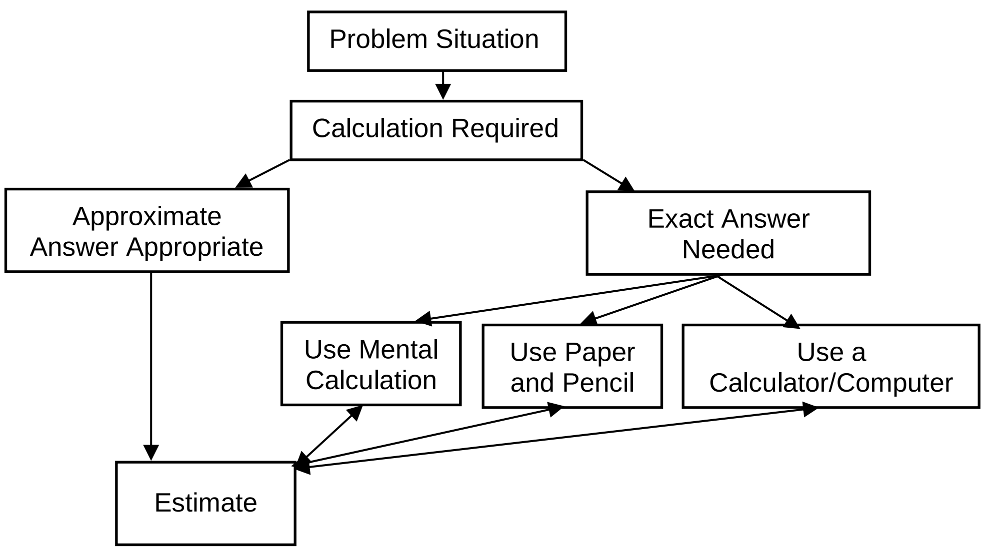
For instance, opportunities should be created frequently to link mathematics and career opportunities. Students need to become aware of the importance of mathematics and the need for mathematics in many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in further areas of study.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It involves calculation without the use of external memory aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. Even more important than performing computational procedures or using calculators is the greater facility that students need - more than ever before - with estimation and mental mathematics (National Council of Teachers of Mathematics, May 2005). Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001). Mental mathematics “provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers” (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know when to estimate, what strategy to use, and how to use it. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process described below:



(NCTM)

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you. . . ?" or “How could you. . . ?” the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident and cognitive mathematical risk takers.

Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, the following strategies should all become familiar to students:

- using estimation
- guessing and checking
- looking for a pattern
- making an organized list or table
- using a model
- working backwards
- using a formula
- using a graph, diagram, or flow chart
- solving a simpler problem
- using algebra.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyse observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators and computers can be used to

- explore and demonstrate mathematical relationships and patterns;
- organize and display data;
- extrapolate and interpolate;
- assist with calculation procedures as part of solving problems;
- decrease the time spent on computations when other mathematical learning is the focus;
- reinforce the learning of basic facts and test properties;
- develop personal procedures for mathematical operations;
- create geometric displays;
- simulate situations;
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in K-3 to enrich learning, it is expected that students will meet all outcomes without the use of technology.

Visualization [V]

Visualization involves thinking in pictures and images, and the ability to perceive, transform, and recreate different aspects of the visual-spatial world. The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3D objects and 2D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate, and knowledge of several estimation strategies (Shaw & Cliatt, 1989).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

➢ The Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics which are woven throughout this document. These components include change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4;
- an arithmetic sequence, with first term 4 and a common difference of 2; or
- a linear function with a discrete domain.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state, and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180^0.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations, or the angle sums of polygons.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (*The Primary Program*, B.C., 2000, p. 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences, and use benchmarks and referents. This results in students who are computationally fluent, and flexible and intuitive with numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with and understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking that is foundational for working with more abstract mathematics in higher grades.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collecting and analysing of data, and describing relationships visually, symbolically, orally, or in written form.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to interpret representations of 2D shapes and 3D objects and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 2D shapes and 3D objects.

Spatial sense offers a way to interpret and reflect on the physical environment and its 3D or 2D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to use dimensions and make predictions about the results of changing dimensions.

- Knowing the dimensions of an object enables students to communicate about the object and create representations.
- The volume of a rectangular solid can be calculated from given dimensions.
- Doubling the length of the side of a square increases the area by a factor of four.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately

Contexts for Learning and Teaching

The Prince Edward Island mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice:

- Mathematics learning is an active and constructive process.
- Learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates.
- Learning is most likely to occur in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Young children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Their mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Initial problem solving and reasoning skills are fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students, and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, and symbolic representations of mathematics.

The learning environment should value and respect the experiences and ways of thinking of all students, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must be encouraged that it is acceptable to solve problems in different ways and realize that solutions may vary.

➢ Homework

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding, and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should replace some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic.

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics before teaching them in the classroom. Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a teacher can give a parent clearer understanding of the mathematics curriculum and of the child's progress in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

➢ Diversity in Student Needs

Every class has students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students. Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters, but should be designed to help all students (whether strong, weak or average) to reach their highest potential. There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson from which all students come away with a better understanding of what the solution to an equation really means.

➢ Gender and Cultural Equity

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean not only that enrolments of students of both genders and various cultural backgrounds in public school mathematics courses should reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

➢ Mathematics for EAL Learners

The Prince Edward Island mathematics curriculum is committed to the principle that learners of English as an additional language (EAL) should be full participants in all aspects of mathematics education. English deficiencies and cultural differences must not be barriers to full participation. All students should study a comprehensive mathematics curriculum with high-quality instruction and co-ordinated assessment.

The *Principles and Standards for School Mathematics* (NCTM, 2000) emphasizes communication "as an essential part of mathematics and mathematics education" (p.60). The *Standards* elaborate that all students, and EAL learners in particular, need to have opportunities and be given encouragement and support for speaking, writing, reading, and listening in mathematics classes. Such efforts have the potential to help EAL learners overcome barriers and will facilitate “communicating to learn mathematics and learning to communicate mathematically” (NCTM, p.60).

To this end,

- schools should provide EAL learners with support in their dominant language and English language while learning mathematics;
- teachers, counsellors, and other professionals should consider the English-language proficiency level of EAL learners as well as their prior course work in mathematics;
- the mathematics proficiency level of EAL learners should be solely based on their prior academic record and not on other factors;
- mathematics teaching, curriculum, and assessment strategies should be based on best practices and build on the prior knowledge and experiences of students and on their cultural heritage;
- the importance of mathematics and the nature of the mathematics program should be communicated with appropriate language support to both students and parents;
- to verify that barriers have been removed, educators should monitor enrolment and achievement data to determine whether EAL learners have gained access to, and are succeeding in, mathematics courses.

➢ Education for Sustainable Development

Education for sustainable development (ESD) involves incorporating the key themes of sustainable development - such as poverty alleviation, human rights, health, environmental protection, and climate change - into the education system. ESD is a complex and evolving concept and requires learning about these key themes from a social, cultural, environmental, and economic perspective, and exploring how those factors are interrelated and interdependent.

With this in mind, it is important that all teachers, including mathematics teachers, attempt to incorporate these key themes in their subject areas. One tool that can be used is the searchable on-line database *Resources for Rethinking*, found at **http://r4r.ca/en**. It provides teachers with access to materials that integrate ecological, social, and economic spheres through active, relevant, interdisciplinary learning.

Assessment and Evaluation

Assessment and evaluation are essential components of teaching and learning in mathematics. The basic principles of assessment and evaluation are as follows:

- Effective assessment and evaluation are essential to improving student learning.
- Effective assessment and evaluation are aligned with the curriculum outcomes.
- A variety of tasks in an appropriate balance gives students multiple opportunities to demonstrate their knowledge and skills.
- Effective evaluation requires multiple sources of assessment information to inform judgments and decisions about the quality of student learning.
- Meaningful assessment data can demonstrate student understanding of mathematical ideas, student proficiency in mathematical procedures, and student beliefs and attitudes about mathematics.

Without effective assessment and evaluation it is impossible to know whether students have learned, or teaching has been effective, or how best to address student learning needs. The quality of the assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others.

➢ Assessment

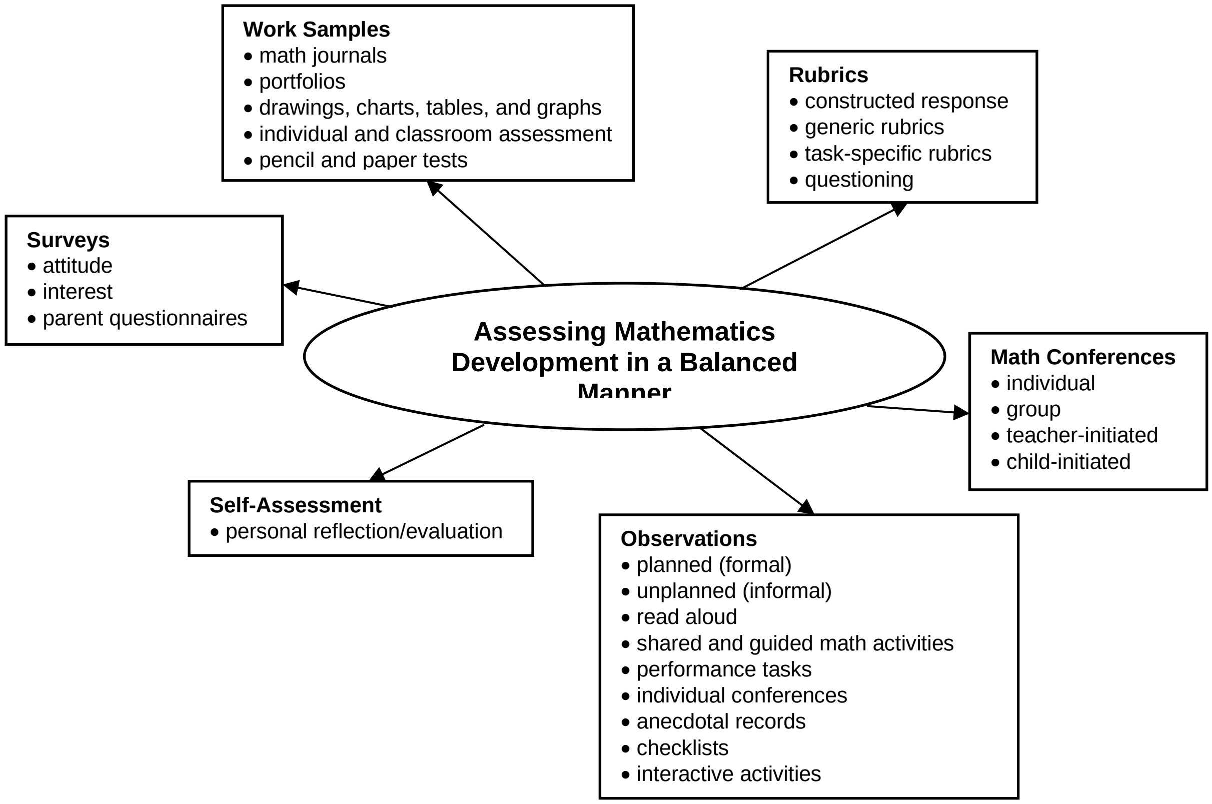
Assessment is the systematic process of gathering information on student learning. To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. Teacher-developed assessments have a wide variety of uses, such as

- providing feedback to improve student learning;
- determining if curriculum outcomes have been achieved;
- certifying that students have achieved certain levels of performance;
- setting goals for future student learning;
- communicating with parents about their children's learning;
- providing information to teachers on the effectiveness of their teaching, the program, and the learning environment;
- meeting the needs of guidance and administration.

A broad assessment plan for mathematics ensures a balanced approach to summarizing and reporting. It should consider evidence from a variety of sources, including

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment.

This balanced approach for assessing mathematics development is illustrated in the diagram on the next page.



There are three interrelated purposes for classroom assessment: assessment *as* learning, assessment *for* learning, and assessment *of* learning. Characteristics of each type of assessment are highlighted below.

Assessment *as* learning is used

- to engage students in their own learning and self-assessment;
- to help students understand what is important in the mathematical concepts and particular tasks they encounter;
- to develop effective habits of metacognition and self-coaching;
- to help students understand themselves as learners - *how* they learn as well as *what* they learn - and to provide strategies for reflecting on and adjusting their learning.

Assessment *for* learning is used

- to gather and use ongoing information in relation to curriculum outcomes in order to adjust instruction and determine next steps for individual learners and groups;
- to identify students who are at risk, and to develop insight into particular needs in order to differentiate learning and provide the scaffolding needed;
- to provide feedback to students about how they are doing and how they might improve;
- to provide feedback to other professionals and to parents about how to support students' learning.

Assessment *of* learning is used

- to determine the level of proficiency that a student has demonstrated in terms of the designated learning outcomes for a unit or group of units;
- to facilitate reporting;
- to provide the basis for sound decision-making about next steps in a student's learning.

Evaluation

Evaluation is the process of analysing, reflecting upon, and summarizing assessment information, and making judgments or decisions based upon the information gathered. Evaluation involves teachers and others in analysing and reflecting upon information about student learning gathered in a variety of ways.

This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work;
- synthesizing information from multiple sources;
- weighing and balancing all available information;
- using a high level of professional judgment in making decisions based upon that information.

Reporting

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning. Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support. Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes, and phone calls.

Guiding Principles

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration, and use of assessments must be followed. The document *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles, as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes, or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate, and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount;
- assessment informs teaching and promotes learning;
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes;
- assessment is fair and equitable to all students and involves multiple sources of information.

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he/she knows and can do.

Structure and Design of the Curriculum Guide

The learning outcomes in the Prince Edward Island mathematics curriculum are organized into four strands across the grades K-9. They are Number, Patterns and Relations, Shape and Space, and Statistics and Probability. These strands are further subdivided into sub-strands, which are the general curriculum outcomes (GCOs). They are overarching statements about what students are expected to learn in each strand or sub-strand from grades K-9.

Strand	General Curriculum Outcome (GCO)
Number (N)	**Number:** Develop number sense.
Patterns and Relations (PR)	**Patterns**: Use patterns to describe the world and solve problems.
	Variables and Equations: Represent algebraic expressions in multiple ways.
Shape and Space (SS)	**Measurement**: Use direct and indirect measure to solve problems.
	3D Objects and 2D Shapes: Describe the characteristics of 3D objects and 2D shapes, and analyse the relationships among them.
	Transformations: Describe and analyse position and motion of objects and shapes.
Statistics and Probability (SP)	**Data Analysis**: Collect, display, and analyse data to solve problems.
	Chance and Uncertainty: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Each general curriculum outcome is then subdivided into a number of specific curriculum outcomes (SCOs). Specific curriculum outcomes are statements that identify the specific skills, understandings, and knowledge students are required to attain by the end of a given grade.

Finally, each specific curriculum outcome has a list of achievement indicators that are used to determine whether students have met the corresponding specific outcome.

In this curriculum guide, each specific curriculum outcome (SCO) is presented in a two-page format, and includes the following information:

- its corresponding strand and general curriculum outcome;
- the scope and sequence of the specific curriculum outcome(s) from grades eight to ten (MAT421A) which correspond to this SCO;
- the specific curriculum outcome, with a list of achievement indicators;
- a list of the sections in *MathLinks 9* which address the SCO, with specific achievement indicators highlighted in brackets;
- an elaboration for the SCO.

NUMBER

SPECIFIC CURRICULUM OUTCOMES

9.N1 – Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:
- **representing repeated multiplication using powers;**
- **using patterns to show that a power with an exponent of zero is equal to one;**
- **solving problems involving powers.**

9.N2 – Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.

9.N3 – Demonstrate an understanding of rational numbers by:
- **comparing and ordering rational numbers;**
- **solving problems that involve arithmetic operations on rational numbers.**

9.N4 – Explain and apply the order of operations, including exponents, with and without technology.

9.N5 – Demonstrate an understanding of perfect square and square root, concretely, pictorially and symbolically (limited to whole numbers).

9.N6 – Determine the square root of positive rational numbers that are perfect squares.

9.N7 – Determine an approximate square root of positive rational numbers that are non-perfect squares.

Grade 9 – Strand: Number (N)

GCO: Develop number sense.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
	9.N1 Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by: • representing repeated multiplication using powers; • using patterns to show that a power with an exponent of zero is equal to one; • solving problems involving powers.	**AN3** Demonstrate an understanding of powers with integral and rational exponents.

SCO: **9.N1 – Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:**

- **representing repeated multiplication using powers;**
- **using patterns to show that a power with an exponent of zero is equal to one;**
- **solving problems involving powers.**

[C, CN, PS, R]

Students who have achieved this outcome should be able to:

A. demonstrate the differences between the exponent and the base by building models of a given power, such as 2^3 and 3^2;

B. explain, using repeated multiplication, the difference between two given powers in which the exponent and base are interchanged, *e.g.*, 10^3 and 3^{10};

C. express a given power as a repeated multiplication;

D. express a given repeated multiplication as a power;

E. explain the role of parentheses in powers by evaluating a given set of powers, *e.g.*, $(-2)^4$, (-2^4), and -2^4;

F. demonstrate, using patterns, that a^0 is equal to 1 for a given value of a $(a \neq 0)$; and

G. evaluate powers with integral bases (excluding base 0) and whole number exponents.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.1 (A B C D E G)

3.2 (C D E F G)

3.3 (D E G)

3.4 (A D)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: 9.N1 – Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:

- **representing repeated multiplication using powers;**
- **using patterns to show that a power with an exponent of zero is equal to one;**
- **solving problems involving powers.**

[C, CN, PS, R]

Elaboration

Students have had experience with perfect squares in relation to area in grade eight. The terms *exponent*, *base*, and *power* (an expression made up of an exponent and a base) are used differently in different resources. For example, the power 6^4 (where 6 is the base and 4 is the exponent), may be described as "six to the power of four", "the fourth power of six", or as "six raised to the power of four" in various textbooks. For consistency and understanding, teachers are asked to use "six to the exponent of four", or "six to the fourth".

Students should be able to link the term *squared* with a 2D area model and *cubed* with a 3D volume model. This will help connect units for area and volume (*e.g.*, square centimetres as cm^2, cubic metres as m^3) to measurement and geometry. It should be emphasized that sometimes the same number can be expressed in multiple ways using powers (*e.g.*, $64 = 8^2$, 4^3 or 2^6).

Students should be able to express 3^5 as $3 \times 3 \times 3 \times 3 \times 3$ and 5^3 as $5 \times 5 \times 5$. Students should also be able to explain the role of parentheses in powers by evaluating a given set of powers. For example:

$$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16, \text{ where the base is } -2$$

$$\left(-2^4\right) = -(2 \times 2 \times 2 \times 2) = -16, \text{ where the base is } 2$$

$$-2^4 = -(2 \times 2 \times 2 \times 2) = -16, \text{ where the base is } 2$$

Students should also be able to demonstrate that $a^0 = 1$, $a \neq 0$, for a given value of *a*, using patterns.

Grade 9 – Strand: Number (N)

GCO: Develop number sense.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
	9.N2 Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.	**AN3** Demonstrate an understanding of powers with integral and rational exponents.

SCO: 9.N2 – Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents. [C, CN, PS, R, T]

Students who have achieved this outcome should be able to:

A. explain, using examples, the exponent laws of powers with integral bases (excluding base 0) and whole number exponents:

- $\left(a^m\right)\left(a^n\right)=a^{m+n}$;
- $a^m \div a^n = a^{m-n}$, $m > n$;
- $\left(a^m\right)^n = a^{mn}$;
- $\left(ab\right)^m = a^m b^m$;
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $b \neq 0$.

B. evaluate a given expression by applying the exponent laws;

C. determine the sum of two given powers, *e.g.*, 5^2+5^3, and record the process;

D. determine the difference of two given powers, *e.g.*, 4^3-4^2, and record the process; and

E. identify the error(s) in a given simplification of an expression involving powers.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.2 (A B E)

3.3 (A B C D E)

3.4 (A B)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: 9.N2 – Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents. [C, CN, PS, R, T]

Elaboration

The primary focus at grade nine should be on the development of an understanding of the exponent laws of powers with integral bases (except base 0) and whole number exponents. Emphasis on attaching names to the laws should not be the focus of instruction. Rather, students' understanding and ability to apply the laws is essential.

Whenever possible, instruction should be designed so that students discover rules and relationships and are able to verify their discoveries. Otherwise, students may get the impression that the rules of mathematics are no more than "tricks." At this level, practice should involve numerical bases only (extensions to literal bases will be addressed in grade ten).

A clear understanding of the following exponent laws should be developed:

$$\left(a^m\right)\left(a^n\right) = a^{m+n}$$

$$a^m \div a^n = a^{m-n},\ m > n$$

$$\left(a^m\right)^n = a^{mn}$$

$$\left(ab\right)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n},\ b \neq 0$$

When questions involve the sum and difference of powers, the order of operations should be emphasized: *e.g.*, $6^5 + 6^2 \neq 6^7$. In the simplification of expressions involving powers, students should be able to identify and explain the error(s): *e.g.*, $\left(2^3\right)^2 \neq 2^5$ or $5^3 \times 5^4 \neq 5^{12}$. Expressions should be simplified as far as possible before they are evaluated, and before calculators are used.

Grade 9 – Strand: Number (N)

GCO: Develop number sense.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
8.N2 Demonstrate an understanding of ratio and rate. **8.N3** Solve problems that involve rates, ratios and proportional reasoning. **8.N4** Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences). **8.N5** Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.	**9.N3** Demonstrate an understanding of rational numbers by: • comparing and ordering rational numbers; • solving problems that involve arithmetic operations on rational numbers.	**AN2** Demonstrate an understanding of irrational numbers by: • representing, identifying and simplifying irrational numbers; • ordering irrational numbers.

SCO: **9.N3 – Demonstrate an understanding of rational numbers by:**

- **comparing and ordering rational numbers;**
- **solving problems that involve arithmetic operations on rational numbers.**

[C, CN, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. order a given set of rational numbers, in fraction and decimal form, by placing them on a number line, *e.g.*, $\frac{3}{5}, -0.666\ldots, 0.5, -\frac{5}{8}$;

B. identify a rational number that is between two given rational numbers; and

C. solve a given problem involving operations on rational numbers in fraction form and decimal form.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.1 (A B)

2.2 (C)

2.3 (C)

2.4 (C)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: 9.N3 – Demonstrate an understanding of rational numbers by:

- **comparing and ordering rational numbers;**
- **solving problems that involve arithmetic operations on rational numbers.**

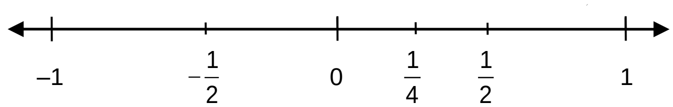
[C, CN, PS, R, T, V]

Elaboration

A rational number is any number that can be written as a fraction or a ratio of two integers, $\frac{a}{b}$, where *b* is never zero. Students have experience with ratios, integers, positive decimals and fraction operations in grades seven and eight. Negative fraction operations will be introduced in grade nine. A review of integer operations will be necessary. The placement of a negative sign in a fraction will be an extension of what students have learned in the past. It is important for students to understand that $\frac{6}{-2}$, $\frac{-6}{2}$ and $-\frac{6}{2}$ are all equivalent fractions. This becomes apparent when the division is completed and all fractions equal –3, regardless of where the negative sign is placed.

Comparing and ordering rational numbers largely draws upon students' number sense. Strategies for ordering numbers should include the following:

- understanding that a negative number is always less than a positive number;
- developing a number line with zero, marking the switch from positive to negative numbers, and with positioning of positive and negative benchmark fractions without conversion to decimals;



- comparing fractions with the same denominator, with unlike denominators, and with the same numerator; students should develop a variety of strategies to compare fractions in addition to creating equivalent denominators;
- identifying fractions between any two given fractions, or decimals between any two decimals, such as between each of the following pairs of numbers: 0.3 and 0.4, $\frac{1}{3}$ and $\frac{1}{2}$, $-\frac{1}{2}$ and $-\frac{1}{3}$.

Mental math and estimation should be used when solving these problems. In this context, calculators could be used as a means of verifying answers.

Grade 9 – Strand: Number (N)

GCO: Develop number sense.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
	9.N4 Explain and apply the order of operations, including exponents, with and without technology.	

SCO: 9.N4 – Explain and apply the order of operations, including exponents, with and without technology. [PS, T]

Students who have achieved this outcome should be able to:

A. solve a given problem by applying the order of operations without the use of technology;

B. solve a given problem by applying the order of operations with the use of technology; and

C. identify the error in applying the order of operations in a given incorrect solution.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.3 (A B C)

3.4 (A B)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: 9.N4 – Explain and apply the order of operations, including exponents, with and without technology.
[PS, T]

Elaboration

The first time order of operations was taught as a specific outcome was in grade six. However, this is practised in grades seven and eight when solving problems involving a variety of operations with integers, positive decimals and fractions. In grade nine, they will extend the rules of order of operations to exponents and to negative rational numbers.

The order of operations is:

1. **B**rackets
2. **E**xponents
3. **D**ivide and **M**ultiply, in order from left to right
4. **A**dd and **S**ubtract, in order from left to right

The acronym **BEDMAS** is often used to help students remember the order of operations.

It is important for students to demonstrate their understanding of these rules, with and without the use of calculators. Student should demonstrate a competence in evaluating expressions that include fractions, fractions squared or cubed, decimals, and negative integers.

Calculators can be used as a tool to check work, in order to gain an understanding of the correct sequence of keys for each student's personal calculator. However the same sequence may be interpreted differently by another calculator. An exploration of this variation could offer an opportunity to develop a better understanding of the correct order of operations. It is important for students to know how their personal calculators process the input and that they are able to apply this knowledge to new situations.

As an example, students can enter the expression $2+3\times 4$ into their calculators. If a particular calculator gives an answer of 14, then it correctly applies the order of operations. However, if it gives an answer of 20, that student will know that his or her calculator does not apply the correct order of operations. In that case, ensure that students have checked their calculators for the proper keying sequence that models the correct order of operations. Some calculators will require additional bracketing to produce the correct answer.

As an indication of understanding, students should be given steps towards an incorrect solution to a problem and be able to identify the step at which the error occurred.

Grade 9 – Strand: Number (N)

GCO: Develop number sense.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
	9.N5 Demonstrate an understanding of perfect square and square root, concretely, pictorially and symbolically (limited to whole numbers).	**AN1** Demonstrate an understanding of factors of whole numbers by determining the: • prime factors; • greatest common factor; • least common multiple; • square root; • cube root.

SCO: **9.N5 – Demonstrate an understanding of perfect square and square root, concretely, pictorially and symbolically (limited to whole numbers).** **[C, CN, R, V]**

Students who have achieved this outcome should be able to:

A. represent a given perfect square as a square region using materials, such as grid paper or square shapes;

B. determine the factors of a given perfect square, and explain why one of the factors is the square root and the others are not;

C. determine whether or not a given number is a perfect square using materials and strategies, such as square shapes, grid paper or prime factorization, and explain the reasoning;

D. determine the square root of a given perfect square and record it symbolically; and

E. determine the square of a given number.

Section(s) in **MathLinks 8** *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

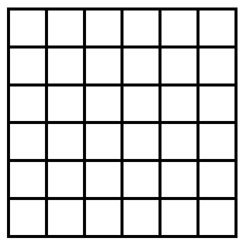
3.1 (A B C D E)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: 9.N5 – Demonstrate an understanding of perfect square and square root, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]

Elaboration

Students should be able to model perfect squares (any whole number squared) and square roots through the use of color tiles or grid paper. They should make a link between these concrete or pictorial representations of square roots and their numerical representations. In the figure below, students should be encouraged to view the area as a perfect square, and either dimension of the square as the square root.



Students should be able to recognize automatically each of the perfect squares from 1 to 144. It is also valuable to bring out the patterns that emerge from a list of perfect squares; that is, students should recognize that the differences between the perfect squares increase in a consistent way as shown in the pattern below:

1 4 9 16 25 36 49

3 5 7 9 11 13

In working with patterns, they should also be exposed to, and predict, other perfect squares. Prime factorization is a method used to find the square root of perfect squares. This will build on what students learned in grade six on prime factors and factor trees. For example, consider $\sqrt{144}$:

Since $144 = 2\times 72$

$= 2\times 2\times 36$

$= 2\times 2\times 6\times 6$ [This process could be stopped at this point if students recognize this as 12×12: $(2\times 6)\times(2\times 6)$.]

$= 2\times 2\times 2\times 3\times 2\times 3$

$= (2\times 2\times 3)\times(2\times 2\times 3)$ [Group factors into two equal groups.]

$= 12\times 12$, therefore, $\sqrt{144} = 12$.

Grade 9 – Strand: Number (N)

GCO: Develop number sense.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
	9.N6 Determine the square root of positive rational numbers that are perfect squares.	**AN1** Demonstrate an understanding of factors of whole numbers by determining the: • prime factors; • greatest common factor; • least common multiple; • square root; • cube root.

SCO: 9.N6 – Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T]

Students who have achieved this outcome should be able to:

A. determine whether or not a given rational number is a square number and explain the reasoning;

B. determine the square root of a given positive rational number that is a perfect square;

C. identify the error made in a given calculation of a square root, (*e.g.*, Is 3.2 the square root of 6.4?); and

D. determine a positive rational number given the square root of that positive rational number.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.4 (A B C D)

[C]	Communication	**[ME]**	Mental Mathematics	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections		and Estimation	**[R]**	Reasoning	**[V]**	Visualization

SCO: 9.N6 – Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T]

Elaboration

Extending the study of square roots of whole numbers, students learn the square root of positive rational numbers that are perfect squares, including whole numbers, fractions and decimals. Mathematicians use the symbol $\sqrt{\ }$ to represent only positive roots, so the solution to $\sqrt{25}$ is 5, which is called the *principal square root*. However, when solving an equation such as $x^2 = 4$, there are two solutions, +2 and –2, since both solutions satisfy the equation:

$$x^2 = 4$$
$$x = \pm\sqrt{4}$$
$$x = \pm 2$$

Students should learn whole number perfect squares to 400 and be able to determine perfect squares beyond 400 through guess and test, using estimation strategies and/or prime factorization. For example, if a student knows that $\sqrt{144} = 12$, and that $\sqrt{400} = 20$, they could estimate that $\sqrt{256}$ lies somewhere between 12 and 20.

Fraction and decimal square roots will all be variations of whole number perfect squares. For example, students will be asked to find $\sqrt{\frac{36}{25}}$, $\sqrt{0.25}$ and $\sqrt{1.44}$. Students should also be able to explain why 25 and 0.25 are perfect squares, but 2.5 is not.

Students should be able to determine a number given its square root. For example, if the square root of a number is 0.7, the number is 0.49. This relates to the fact that squares and square roots are inverse operations, a concept which should be explored. If a student finds the square root of a number and then squares it, he or she will end up where they started.

Grade 9 – Strand: Number (N)

GCO: Develop number sense.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
	9.N7 Determine an approximate square root of positive rational numbers that are non-perfect squares.	**AN1** Demonstrate an understanding of factors of whole numbers by determining the: • prime factors; • greatest common factor; • least common multiple; • square root; • cube root.

SCO: **9.N7 – Determine an approximate square root of positive rational numbers that are non-perfect squares. [C, CN, PS, R, T]**

Students who have achieved this outcome should be able to:

A. estimate the square root of a given rational number that is not a perfect square using the roots of perfect squares as benchmarks;

B. determine an approximate square root of a given rational number that is not a perfect square using technology, *e.g.*, calculator, computer;

C. explain why the square root of a given rational number as shown on a calculator may be an approximation; and

D. identify a number with a square root that is between two given numbers.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

2.4 (A B C D)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: 9.N7 – Determine an approximate square root of positive rational numbers that are non-perfect squares. [C, CN, PS, R, T]

Elaboration

Students will develop a greater intuitive understanding of square root through practicing estimation skills. Students will be required to estimate the square root of rational numbers in fraction and decimal form.

Starting with whole numbers, students should use benchmarks (roots of perfect square numbers) to identify between which two whole numbers the square root will fall and to which whole number it is closer. For example, students should know that the square root of 22 is between 4 and 5, and that it is closer to 5. Given a choice, students should also realize that 22 will be closer to 4.7 than to 4.2. It is very important to emphasize the difference between an exact square root and a decimal approximation. The square root of any non-perfect square will be an irrational number (any number that cannot be converted to the form $\frac{a}{b}$, or a non-terminating, non-repeating decimal). Regardless of the number of decimal places retained in an irrational number, it is still an approximation (*e.g.*, π = 3.1416).

Again, students will use benchmark perfect squares to help with their estimates using various strategies for rational numbers. For example, $\sqrt{0.79}$ is approximately equal to $\sqrt{0.81}$, which is equal to 0.9, so $\sqrt{0.79} \approx 0.9$. Students should also understand that the answer is a little less than 0.9.

Fractions can be addressed in a similar manner in a couple of different ways. For example, $\sqrt{\frac{8}{15}}$ is approximately equal to $\sqrt{\frac{9}{16}}$, which is equal to $\frac{3}{4}$, so $\sqrt{\frac{8}{15}} \doteq \frac{3}{4}$. Another approach that could be used uses the fact that $\frac{8}{15}$ is a little more than $\frac{1}{2}$, which equals 0.5. Since $\sqrt{0.5}$ is approximately equal to $\sqrt{0.49}$, which is equal to 0.7, then $\sqrt{\frac{8}{15}} \doteq 0.7$.

Please note that $\doteq$ and $\approx$ may be both used to symbolize “approximately equal to.”

PATTERNS AND RELATIONS

SPECIFIC CURRICULUM OUTCOMES

PR1 – Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.

PR2 – Model and solve problems using linear equations of the form:

- $ax = b$;
- $\frac{x}{a} = b,\ a \neq 0$;
- $ax + b = c$;
- $\frac{x}{a} + b = c,\ a \neq 0$;
- $ax = b + cx$;
- $a(x + b) = c$;
- $ax + b = cx + d$;
- $a(bx + c) = d(ex + f)$;
- $\frac{a}{x} = b,\ x \neq 0$

where a, b, c, d, e and f are rational numbers.

PR3 – Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.

PR4 – Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).

PR5 – Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2).

PR6 – Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.

Grade 9 – Strand: Patterns and Relations (PR)

GCO: Use patterns to describe the world and solve problems.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
8.PR1 Graph and analyse two-variable linear relations.	**9.PR1** Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.	**RF4** Describe and represent linear relations, using: • words; • ordered pairs; • tables of values; • graphs; • equations. **RF8** Represent a linear function using function notation.

SCO: **9.PR1 – Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution. [C, CN, PS, R, V]**

Students who have achieved this outcome should be able to:

A. write an expression representing a given pictorial, oral or written pattern;

B. write a linear equation to represent a given context;

C. describe a context for a given linear equation;

D. solve, using a linear equation, a given problem that involves pictorial, oral and written linear patterns; and

E. write a linear equation representing the pattern in a given table of values and verify the equation by substituting values from the table.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

6.1 (A B C D E)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: 9.PR1 – Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution. [C, CN, PS, R, V]

Elaboration

Students have been exposed to patterns through the interpretation of graphs of linear relations. From a pictorial pattern, students should be able to identify and write the pattern rule and create a table of values in order to write an expression to represent the situation. When an oral or written pattern is given, students should be able to write an expression directly from that pattern.

Linear expressions have both a variable value and a constant value. This connection is seen in situations involving membership fees, where there is an initial fee (constant value) and a usage fee (variable value). It is important to make a clear distinction between the two. It is also necessary to describe a context represented by a given linear equation.

When students are looking at a table of values, such as the following,

Term Number (*n*)	1	2	3	4	5
Term (*t*)	2	8	14	20	26

they should look at the pattern and recognize a constant increase or decrease (here an increase of 6) between the values. Students should recognize that multiplying the term number, n, by 6 always results in four more than the associated term, t. Therefore, they will need to subtract 4 from $6n$. As an equation, the pattern is represented by $t = 6n - 4$. Students should verify their equation by substituting values from the table (for example, $n = 5$, $t = 26$). Students should use their equation to solve for any value of n or t.

Students need to be able to transition between given information, whether it is presented as a pictorial pattern, table of values, algebraic expressions, a graph, a linear relation or a set of ordered pairs.

Grade 9 – Strand: Patterns and Relations (PR)

GCO: Represent algebraic expressions in multiple ways.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
8.PR2 Model and solve problems using linear equations of the form: • $ax = b$; • $\frac{x}{a} = b,\ a \neq 0$; • $ax + b = c$; • $\frac{x}{a} + b = c,\ a \neq 0$; • $a(x + b) = c$ concretely, pictorially and symbolically, where *a*, *b* and *c* are integers.	**9.PR2** Model and solve problems using linear equations of the form: • $ax = b$; • $\frac{x}{a} = b,\ a \neq 0$; • $ax + b = c$; • $\frac{x}{a} + b = c,\ a \neq 0$; • $ax = b + cx$; • $a(x + b) = c$; • $ax + b = cx + d$; • $a(bx + c) = d(ex + f)$; • $\frac{a}{x} = b,\ x \neq 0$ where *a*, *b*, *c*, *d*, *e* and *f* are rational numbers.	**RF8** Represent a linear function using function notation. **RF9** Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

SCO: 9.PR2 – Model and solve problems using linear equations of the form:

- $\boldsymbol{ax = b}$;
- $\boldsymbol{\frac{x}{a} = b,\ a \neq 0}$;
- $\boldsymbol{ax + b = c}$;
- $\boldsymbol{\frac{x}{a} + b = c,\ a \neq 0}$;
- $\boldsymbol{ax = b + cx}$;
- $\boldsymbol{a(x + b) = c}$;
- $\boldsymbol{ax + b = cx + d}$;
- $\boldsymbol{a(bx + c) = d(ex + f)}$;
- $\boldsymbol{\frac{a}{x} = b,\ x \neq 0}$

where *a*, *b*, *c*, *d*, *e* and *f* are rational numbers. [C, CN, PS, V]

Students who have achieved this outcome should be able to:

A. model the solution of a given linear equation using concrete or pictorial representations, and record the process;

B. determine, by substitution, whether a given rational number is a solution to a given linear equation;

C. solve a given linear equation symbolically;

D. identify and correct an error in a given incorrect solution of a linear equation;

E. represent a given problem using a linear equation; and

F. solve a given problem using a linear equation and record the process.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

8.1 (A B C E F)

8.2 (A B C D E F)

8.3 (A B C D E F)

8.4 (A B C D E F)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: 9.PR2 – Model and solve problems using linear equations of the form:

- $ax = b$;
- $\frac{x}{a} = b,\ a \neq 0$;
- $ax + b = c$;
- $\frac{x}{a} + b = c,\ a \neq 0$;
- $ax = b + cx$;
- $a(x + b) = c$;
- $ax + b = cx + d$;
- $a(bx + c) = d(ex + f)$;
- $\frac{a}{x} = b,\ x \neq 0$

where *a*, *b*, *c*, *d*, *e* and *f* are rational numbers. [C, CN, PS, V]

Elaboration

In grade eight, students have experience solving one and two-step equations in the following forms:

$$ax = b; \quad \frac{x}{a} = b,\ a \neq 0; \quad ax + b = c; \quad \frac{x}{a} + b = c,\ a \neq 0; \quad a(x + b) = c.$$

A review of the various informal methods used to solve equations developed in grades seven and eight may be necessary. These could include the use of algebra tiles, inspection, or systematic trials (guess and test).

In grade nine, students will continue to solve equations which include integers and rational numbers, when the variable is found on both sides of the equal sign or found in the denominator, and when more than two steps are required to solve the equation.

In problem-solving situations, students should be aware that once they acquire a solution, it can be checked to see if it is correct by substitution into the original equation.

Proper use of vocabulary should be modelled. The following terms should be used where appropriate: *relationship*, *equality*, *algebraic equation*, *distributive property*, *like terms*, *balancing*, *the zero principle*, *the elimination process*, *isolating variables*, *coefficient*, *constant*, *equation* and *expression*. In this way, students will become more comfortable with the mathematical vocabulary and not find it so intimidating or confusing.

Students should be able to model how the solution of a given linear equation is determined using concrete or pictorial representations. After adequate practice and understanding, students should be able to transfer the model to pencil and paper.

Grade 9 – Strand: Patterns and Relations (PR)

GCO: Represent algebraic expressions in multiple ways.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
	9.PR3 Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.	**RF5** Determine the characteristics of the graphs of linear relations, including the: • intercepts; • slope; • domain; • range.

SCO: **9.PR3 – Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.** **[C, CN, PS, R, V]**

Students who have achieved this outcome should be able to:

A. translate a given problem into a single variable linear inequality using the symbols $\geq$, $>$, $<$ or $\leq$;

B. determine if a given rational number is a possible solution of a given linear inequality;

C. generalize and apply a rule for adding or subtracting a positive or negative number to determine the solution of a given inequality;

D. generalize and apply a rule for multiplying or dividing by a positive or negative number to determine the solution of a given inequality;

E. solve a given linear inequality algebraically and explain the process orally or in written form;

F. compare and explain the process for solving a given linear equation to the process for solving a given linear inequality;

G. graph the solution of a given linear inequality on a number line;

H. compare and explain the solution of a given linear equation to the solution of a given linear inequality;

I. verify the solution of a given linear inequality using substitution for multiple elements in the solution; and

J. solve a given problem involving a single variable linear inequality and graph the solution.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

9.1 (A B G)

9.2 (A B C D E F G H J)

9.3 (A E I J)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: 9.PR3 – Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context. [C, CN, PS, R, V]

Elaboration

Solving inequalities is a new concept in grade nine. Students will be working with linear inequalities at this point. An inequality is defined as a mathematical sentence that compares two expressions that may or may not be equal. Students need to realize that this type of problem may have many solutions rather than just one as with most linear equations.

Students will build on their previous knowledge of solving linear equations and expand to inequalities where the operation rules are the same, with the exception of multiplying or dividing both sides of the inequality by a negative number. This will result in the inequality sign changing orientation.

Emphasis should be placed on having students graph their solutions on a number line to understand clearly what the answer represents, that is, a set of values instead of a single solution. These solutions can be represented pictorially as a set of values on a number line.

Where possible, an effort should be made to have students describe a problem or situation as an inequality. These then can be solved and represented on a number line. Many of these problems are real life situations. This is a good opportunity for teachers to discuss with students that there may or may not be limits on these inequalities that are created, depending on the context of the problem. For example, if you are discussing the speed of a vehicle, this will not be negative, so instead of saying $v < 10$, we would have to understand that we mean $0 \le v < 10$.

Grade 9 – Strand: Patterns and Relations (PR)

GCO: Represent algebraic expressions in multiple ways.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
	9.PR4 Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).	

SCO: 9.PR4 – Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V]

Students who have achieved this outcome should be able to:

A. create a concrete model or a pictorial representation for a given polynomial expression;

B. write the expression for a given model of a polynomial;

C. identify the variables, degree, number of terms and coefficients, including the constant term, of a given simplified polynomial expression;

D. describe a situation for a given first degree polynomial expression; and

E. match equivalent polynomial expressions given in simplified form, *e.g.*, $4x - 3x^2 + 2$ is equivalent to $-3x^2 + 4x + 2$.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.1 (A B C D E)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: 9.PR4 – Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V]

Elaboration

There is a significant amount of new vocabulary introduced in this section:

- A *term* is an expression formed from the product of numbers and/or variables, for example, 2, $3x^2$ and $4x$ are all terms.
- A *constant term* is a term that does not have a variable factor.
- A *coefficient* is the number by which the variable is multiplied.
- A *polynomial* is an algebraic expression made up of terms connected by the operations of addition or subtraction.
- The *degree of a term* is the sum of the exponents on the variables in a single term. For example, the degree of $4xy^2$ is 3. A variable with no exponent showing is understood to have an exponent of one.
- The *degree of a polynomial* is the highest degree of any term in a polynomial.
- All expressions with one or more terms are called *polynomials*. Some polynomials are named by the number of terms they contain, for example *monomial* (one term), *binomial* (two terms) and *trinomial* (three terms).

A review of the models used for linear equations in grade eight could be done here. Students should be familiar with the use of algebra tiles for modelling linear situations from previous grades. The introduction of a tile for x^2 will be needed so that students can represent second degree polynomials.

Students should be comfortable at this stage changing from models and pictorial representations to polynomial expressions, and polynomial expressions back to models and pictorial representation. Rearranging polynomial expressions to show that some expressions are equivalent should also be included.

Grade 9 – Strand: Patterns and Relations (PR)

GCO: Represent algebraic expressions in multiple ways.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
	9.PR5 Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2).	

SCO: **9.PR5 – Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, PS, R, V]**

Students who have achieved this outcome should be able to:

A. model addition of two given polynomial expressions concretely or pictorially and record the process symbolically;

B. model subtraction of two given polynomial expressions concretely or pictorially and record the process symbolically;

C. apply a personal strategy for addition and subtraction of given polynomial expressions, and record the process symbolically;

D. identify equivalent polynomial expressions from a given set of polynomial expressions, including pictorial and symbolic representations; and

E. identify the error(s) in a given simplification of a given polynomial expression.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

5.2 (C D E)

5.3 (A B C)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: 9.PR5 – Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, PS, R, V]

Elaboration

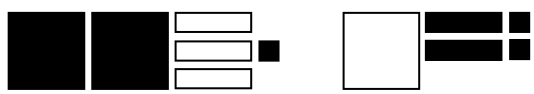
Students were exposed to the notion of a variable in grade five, as algebraic reasoning is a focus of the revised curriculum. As part of the continued development of algebra, students should be given a variety of ways to relate to the symbolism. One connection which may be useful to students relates to measurement situations. For example, can we add $3m+5m^2$ and get 8 of something? Students should realize that the units must be the same in order to add or subtract them. This should enable them to more easily transfer to the concept of like and unlike terms. The use of algebra tiles as models will strengthen this transfer as students visualize the difference between x and x^2 or between x and 1.

Students have also worked with integers and modeled operations with two colour counters, so they have experience with the idea of positive and negative numbers and the zero principle. But first, time should be spent on modeling and identifying like and unlike terms before the addition and subtraction of polynomials is introduced.

It is important that when the addition and subtraction of polynomial expressions are modeled that students record the expressions symbolically at the same time.

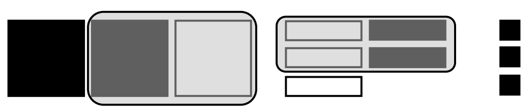
As an example, given the expression:

$\left(2x^2-3x+1\right)+\left(-x^2+2x+2\right)$



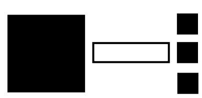
Combine like terms:

$\left(2x^2-x^2\right)+\left(-3x+2x\right)+\left(1+2\right)$



Remove zeros:

x^2-x+3



It is important that students move from the concrete to the pictorial to the symbolic, but initially, either the concrete and symbolic, or the pictorial and symbolic should be done in tandem.

Grade 9 – Strand: Patterns and Relations (PR)

GCO: Represent algebraic expressions in multiple ways.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
	9.PR6 Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.	**AN4** Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically. **AN5** Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

SCO: **9.PR6 – Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically. [C, CN, R, V]**

Students who have achieved this outcome should be able to:

A. model multiplication of a given polynomial expression by a given monomial concretely or pictorially and record the process symbolically;

B. model division of a given polynomial expression by a given monomial concretely or pictorially and record the process symbolically;

C. apply a personal strategy for multiplication and division of a given polynomial expression by a given monomial;

D. provide examples of equivalent polynomial expressions; and

E. identify the error(s) in a given simplification of a given polynomial expression.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

7.1 (A B C D E)

7.2 (A C D E)

7.3 (B C D E)

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

SCO: 9.PR6 – Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically. [C, CN, R, V]

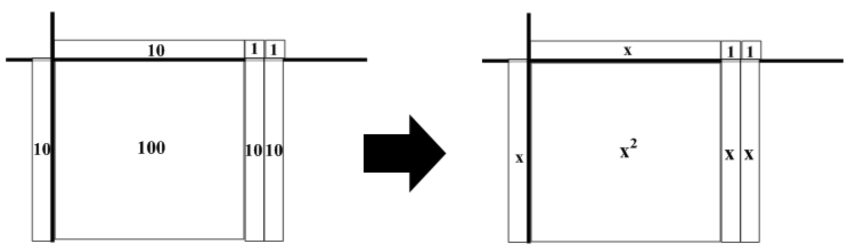
Elaboration

This outcome, dealing with the multiplication and division of polynomial expressions, is restricted to expressions of no greater than second degree, to allow for the effective modeling of both operations. In other words, no part of any expression can have an exponent greater than two. The focus is on developing a deep understanding of the operations.

Students should first explore multiplication and division of a monomial by a monomial, extend to multiplication and division of a polynomial by a number, and then move to multiplication and division of a polynomial by a monomial.

The area model is a powerful model for students to use. They have had experience with the area model in numerical situations, such as modeling one-digit and two-digit whole number multiplication as well as multiplication with fractions. Base ten materials have typically been used for these models so the transfer to algebra tiles should be smooth.

For example, this area model could be used to show 10×12, or $10(10+2)$, as well as $x(x+2)$ or the related division of $\frac{x^2+2x}{x}$.



SHAPE AND SPACE

SPECIFIC CURRICULUM OUTCOMES

9.SS1 – Develop and apply the Pythagorean theorem to solve problems.

9.SS2 – Determine the surface area of composite 3D objects to solve problems.

9.SS3 – Demonstrate an understanding of similarity of triangles.

9.SS4 – Draw and interpret scale diagrams of 2D shapes.

Grade 9 – Strand: Shape and Space (SS)

GCO: Use direct or indirect measurement to solve problems.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
8.SS2 Determine the surface area of: • right rectangular prisms; • right triangular prisms; • right cylinders to solve problems.	**9.SS1** Develop and apply the Pythagorean theorem to solve problems.	**M3** Solve problems, using SI and imperial units, that involve the surface area and volume of 3D objects, including: • right cones; • right cylinders; • right prisms; • right pyramids; • spheres. **M4** Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

SCO: 9.SS1 – Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V]

Students who have achieved this outcome should be able to:

A. model and explain the Pythagorean theorem concretely, pictorially or using technology;

B. explain, using examples, that the Pythagorean theorem applies only to right triangles;

C. determine whether or not a given triangle is a right triangle by applying the Pythagorean theorem;

D. determine the measure of the third side of a right triangle, given the measures of the other two sides, to solve a given problem; and

E. solve a given problem that involves Pythagorean triples, *e.g.,* 3, 4, 5 or 5, 12, 13.

Section(s) in **MathLinks 8** *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

3.2 (A B C E)

3.4 (D)

3.5 (D)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: 9.SS1 – Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V]

Elaboration

Pythagoras of Samos, *c*. 560 BC – *c*. 480 BC, was a Greek philosopher who is credited with providing the first proof of the Pythagorean relationship. It states that the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides. The conventional formula for the Pythagorean relationship, $c^2 = a^2 + b^2$, should be developed through investigations. It is also important for students to recognize that the Pythagorean relationship can be labelled differently from the conventional *a-b-c* notation. The hypotenuse, or the longest side, is *c* and two shorter sides, or legs, are *a* and *b*.

A Pythagorean triple is any set of three whole numbers *a, b* and *c*, for which $a^2 + b^2 = c^2$. It is believed that the Egyptians and other ancient cultures used a 3-4-5 rule $(a = 3, b = 4, c = 5)$ in construction to ensure buildings were square. The 3-4-5 rule allowed them a quick method of establishing a right angle. This method is still used today in construction.

In presenting diagrams of right triangles, it is important to give diagrams of the triangles in various orientations. Students should recognize the hypotenuse as being the side opposite the right angle, regardless of the orientation of the figure. Whenever a triangle has a right angle and two known side lengths, the Pythagorean relationship should be recognized by students. Students should be given experiences with side lengths of triangles that do not make right angle triangles. Students should also be provided with experiences that involve finding the length of the hypotenuse, as well as situations where the hypotenuse and one side is known and the other side is to be found. Also, it is important for students to realize that they can use the Pythagorean relationship when only one side is known, as long as the right triangle is isosceles. Finally, students should be able to use the Pythagorean relationship to determine if three given side lengths are, or are not, the sides of a right triangle. There are many opportunities to use the Pythagorean relationship to solve other problems, such as determining the height of a building or finding the shortest distance across a rectangular field.

Students need to be provided with opportunities to model and explain the Pythagorean theorem concretely, pictorially, and symbolically:

- **Concretely** – by cutting up areas represented by a^2, b^2 and fitting the two areas onto c^2
- **Pictorially** – using grid paper or technology
- **Symbolically** – by confirming that $a^2 + b^2 = c^2$ forms a right triangle

Grade 9 – Strand: Shape and Space (SS)

GCO: Describe the characteristics of 3D objects and 2D shapes, and analyse the relationships among them.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
8.SS2 Determine the surface area of: • right rectangular prisms; • right triangular prisms; • right cylinders to solve problems.	**9.SS2** Determine the surface area of composite 3D objects to solve problems.	**M3** Solve problems, using SI and imperial units, that involve the surface area and volume of 3D objects, including: • right cones; • right cylinders; • right prisms; • right pyramids; • spheres.

SCO: **9.SS2 – Determine the surface area of composite 3D objects to solve problems.** **[C, CN, PS, R, V]**

Students who have achieved this outcome should be able to:

A. determine the area of overlap in a given concrete composite 3D object, and explain its effect on determining the surface area (limited to right cylinders, right rectangular prisms and right triangular prisms);

B. determine the surface area of a given concrete composite 3D object (limited to right cylinders, right rectangular prisms and right triangular prisms); and

C. solve a given problem involving surface area.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

1.3 (A B C)

[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

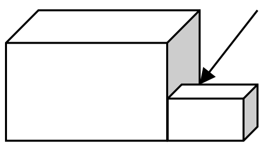
SCO: 9.SS2 – Determine the surface area of composite 3D objects to solve problems. [C, CN, PS, R, V]

Elaboration

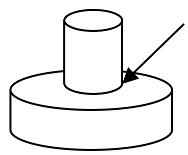
It is important for students to be able to visualize the net of a 3D object to calculate the surface area of that object efficiently. It is important to use concrete materials to help students visualize the relationship between the 2D net and the 3D object. Surface area is the sum of the areas of all the faces of a 3D object. Some students may require a review of strategies for determining the area of 2D shapes. Remind students that square units are used to measure area and surface area.

In grade eight, students had experience calculating surface areas of right rectangular prisms, right triangular prisms, and right cylinders. In grade nine, this is extended to composite objects which are combinations of these same objects. Concrete items can be used to represent the solids when they are combined to form composite 3D objects so students can determine what surface is hidden when the objects are combined. It will become obvious that the area of the overlap of surfaces needs to be subtracted from the original surface area of **each** of the original objects. The objects are all right prisms or cylinders so the hidden surfaces will be symmetrical to an opposite surface that is exposed.

Hidden Surface



Hidden Surface



The students should be presented with examples of scenarios that involve real world applications of the surface area of combined objects such as buildings, containers, packaging or furniture. Some objects may also have voids (missing sections) rather than additions, such as a bookshelf or a pencil holder.

Grade 9 – Strand: Shape and Space (SS)

GCO: Describe the characteristics of 3D objects and 2D shapes, and analyse the relationships among them.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
	9.SS3 Demonstrate an understanding of similarity of triangles.	**M4** Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

SCO: **9.SS3 – Demonstrate an understanding of similarity of triangles.** **[C, CN, PS, R, V]**

Students who have achieved this outcome should be able to:

A. determine if the triangles in a given pre-sorted set are similar and explain the reasoning;

B. determine a missing side length using similar triangles; and

C. solve a given problem using the properties of similar triangles.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.3 (A B C)

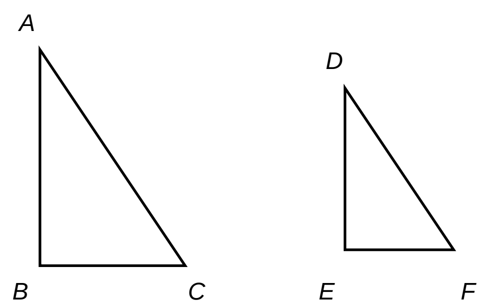
[C] Communication **[CN]** Connections	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving **[R]** Reasoning	**[T]** Technology **[V]** Visualization

SCO: 9.SS3 – Demonstrate an understanding of similarity of triangles. [C, CN, PS, R, V]

Elaboration

The connection between proportional reasoning and the geometric concept of similarity is very important. Similar figures provide a visual representation of proportions, and proportional reasoning enhances the understanding of similarity. To show that triangles are similar, students must compare the ratios of the corresponding side lengths and check that corresponding angles are equal. Corresponding sides are sides that have the same relative position in two geometric figures. When triangles are similar, corresponding angles are congruent and corresponding side lengths are all enlarged or reduced by the same factor (ratio).

Students should understand the relationships between the corresponding sides of similar triangles. That is, if $\Delta ABC \sim \Delta DEF$, then the following ratios are equal: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.



The process of reasoning is necessary for deciding how many trials you need to apply in order to eliminate any doubt that two or more triangles are similar. For example, two corresponding side measures alone cannot be the only test to determine for similarity.

Students should be exposed to a variety of situations, including pairs of similar figures that are varied in their orientations. The properties of similar triangles can be used to find the measures of missing sides and angles. This topic lends itself well to real life situations, such as finding heights of buildings, or distances which are normally difficult to measure directly, such as the distance across a pond. The process of determining the similarity of triangles can also be applied to other polygons.

Please note this outcome should be taught in conjunction with outcome 9.SS4.

Grade 9 – Strand: Shape and Space (SS)

GCO: Describe and analyse position and motion of objects and shapes.

GRADE 8	GRADE 9	GRADE 10 – MAT421A
	9.SS4 Draw and interpret scale diagrams of 2D shapes.	

SCO: **9.SS4 – Draw and interpret scale diagrams of 2D shapes.** **[CN, R, T, V]**

Students who have achieved this outcome should be able to:

A. identify an example in print or electronic media,(*e.g.*, newspapers, the Internet) of a scale diagram and interpret the scale factor;

B. draw a diagram to scale that represents an enlargement or reduction of a given 2D shape;

C. determine the scale factor for a given diagram drawn to scale; and

D. determine if a given diagram is proportional to the original 2D shape and, if it is, state the scale factor.

Section(s) in MathLinks 9 *text that address the specific curriculum outcome with relevant Achievement Indicators in brackets:*

4.1 (A B C) - Limit to simple shapes in which the vertices lie on grid vertices. (See 9.SS4 Supplement on Learn.edu.pe.ca)

4.2 (C D)

[C] Communication	**[ME]** Mental Mathematics and Estimation	**[PS]** Problem Solving	**[T]** Technology
[CN] Connections		**[R]** Reasoning	**[V]** Visualization

SCO: 9.SS4 – Draw and interpret scale diagrams of 2D shapes. [CN, R, T, V]

Elaboration

A scale is a comparison between the actual size of an object and the size of its image therefore, a scale diagram would be a drawing that is similar to the actual figure. These scale diagrams can be either an enlargement or reduction of the actual object depending on the context. If scale factors are bigger than one, this will result in an enlargement whereas if the scale factor is less than one, it is a reduction.

Students have an intuitive sense of shapes that are enlargements or reductions of each other. Students have experienced maps and pictures that have been drawn to scale, and with images produced by photocopiers and computer software. The use of computer software can allow for a great deal of flexibility in the investigation of enlargement and reduction. Methods of using graph paper, scale factors, or protractor and ruler can also be used. It should be noted that when a ratio is used to represent an enlargement or reduction, the format of the ratio is $\text{New} : \text{Original}$. A ratio of $2:1$ means the new figure is an enlargement to twice the size of the original. Likewise, a ratio of $1:3$ means that the new figure is a reduction to $\frac{1}{3}$ of the original, or the original is three times the size of the new figure.

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